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Ring-like Space Constructed from Group-like Space*

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The ring-like space introduced in[1] is more extensive than the topological ring in view of the definition. But, in face, it has not been shown whether the ring-like space is really more extensive than the topological ring. In our study, we first investigate a method of constructing a ring-like space from a group-like space. Then, using this method, ws construct a ring-like space which is not a topological ring.

The definition of the ring-like space[1]. Let R be a topological space, e be a point in it.Let α, $\mu:(R\times R,(e,e))\to(R,e)$ and $i:(R,e)\to(R,e)$ be continuous functions satisfying the ring axioms up to homotopy. Then R is called a ring-like space with respect to those continuous functions.

1°. A Method Let G be a topological space, let a continuous function α: $G\times G\to G$ be homotopy associative, that is, $\alpha\circ(\alpha\times 1)\simeq\alpha\circ(1\times\alpha)$. The disjoint union of G and a point $e\notin G$ is denoted by G^+. G^* has the induced topology, that is, a subset $S\subset G^+$ is open if and only if $S\cap G$ is open in G. If $x,y\in G$, $\alpha^+:G^+\times G^+\to G^+$ is defined by $\alpha^+(x,y)=\alpha(x,y)$and $\alpha^+(x,e)=\alpha^+(e,y)=\alpha^+(e,e)=e$. Then α^+ is a continuous function and also is homotopy associative.

It is by $G^\infty=\bigvee\limits_1^\infty(G^n)^+$ that the disjoint union of all $(G^n)^+$ is denoted except e that is the common point. G^∞ has the induced topology, that is, a subset $S\subset G^\infty$ is open if and only if $S\cap(G^n)^+$ is open in $(G^n)^+$ for each n. By the definition of G^∞, for each $x\in G^\infty$, either $x=e$ or there exists n such that $x\in G^n$, that is, $x=(x_1,\cdots,x_n)$, $x_i\in G$. In G^∞, it is defined that $(x_1,\cdots,x_n)=x=y=(y_1,\cdots,y_m)$ if and only if $m=n$ and $(y_1,\cdots,y_n)$is the rearrangement of $(x_1,\cdots,x_n)$. The quotient space resulted is still denoted by G^∞.

In G^∞, functions β, $\mu:G^\infty\times G^\infty\to G^\infty$ are defined by $\beta(x,y)=(x_1,\cdots,x_n,y_1,\cdots,y_m)$, $\beta(x,e)=x$, $\beta(e,y)=y$, $\beta(e,e)=e$ and $\mu(x,y)=(\alpha^+(x_i,y_j))_{i,j}=(\alpha^+(x_1,y_1),\cdots,\alpha^+(x_1,y_m),$ $\alpha^+(x_2y_1),\cdots,\alpha^+(x_n,y_m))$, for $x=(x_1,\cdots,x_n)\neq e$, $y=(y_1,\cdots,y_m)\neq e$.It is easy to check that β and μ are continuous functions, β is associative and commutative, and μ and

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β are distributive.

Let (x,y), $(x',y')\in G^{\infty}\times G^{\infty}$. It is defined that $(x,y)\sim(x',y')$ if and only if $\beta(x,y')=\beta(y,x')$. It is easily checked that this relation is an equivalence relation. Suppose the quotient space $R=G^{\infty}\times G^{\infty}/\sim$. In R, B, M: $(R\times R,(e,e))\to(R,e)$ and $\triangle:(R,e)\to(R,e)$ are defined as follows:

$$B([x,y],[u,v])=[\beta(x,u),\beta(y,v)],$$

$$M([x,y],[u,v])=[\beta(\mu(x,u),\mu(y,v)),\beta(\mu(x,v),\mu(y,u))],\quad \triangle([x,y])=[y,x],$$

where $[x,y],[u,v]\in R$, the base point of R $e=[x,x]$.

It can be shown that B, M, $\triangle$ are continuous functions and R is a ring-like space with respect to those continuous functions. Thus, a ring-like space R is constructed from a group-like space G.

2° Brief Proof It can be shown that B, M, $\triangle$ are functions by computation.

In order to show that B, M, $\triangle$ are continuous, two familiar facts[2] are used ——the universal property of the cartesian product and the universal property of the quotient. Continuity of B follows from the decomposition.

$$\begin{tikzcd}
G^{\infty}\times G^{\infty}\times G^{\infty}\times G^{\infty} \arrow[r, "1\times T_{23}\times 1"] \arrow[d, "P\times P"] & G^{\infty}\times G^{\infty}\times G^{\infty}\times G^{\infty} \arrow[r, "\beta\times\beta"] & G^{\infty}\times G^{\infty} \arrow[r, "p"] & R \\
R\times R \arrow[rrru, "B"'] & & &
\end{tikzcd}$$

Continuity of M and $\triangle$ follows similarly.

To show that R is a ring-like space with respect to B, M, $\triangle$, it is sufficient to show that $M\circ(M\times 1)\cong M\circ(1\times M)$ rel(e,e,e). The proof is tedious.

3° Example Let $Y=[a,b]$. In loop space $\Omega Y=(Y,y_0)^{(s^1,s_0)}$, $\alpha:\Omega Y\times\Omega Y\to\Omega Y$ is defined by

$$\alpha(x,y)(\sigma)=\begin{cases} x(2\sigma), & 0\leqslant\sigma\leqslant\frac{1}{2}, \\ y(2\sigma-1), & \frac{1}{2}\leqslant\sigma\leqslant 1, \end{cases}$$

then α is continuous[2]. Let $G=\Omega Y$, by 1°, $R=G^{\infty}\times G^{\infty}/\sim$ is a ring-like space.

By Urysohn lemma[3], it is desirable to choose $x,y,u,v,s,t\in G=\Omega([a,b])$ such that $M\circ(M\times 1)\not\cong M\circ(1\times M)$, so R is not a topological ring.

References

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