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WEAK SOLUTIONS OF POPULATION EVOLUTION EQUATIONS AND FORECAST OF AVERAGE LIFE SPAN FOR POPULATION*

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The population evolution equations are theoretical bases of the research on population problems. It is the following boundary value problem:

$$\begin{cases} \dfrac{\partial p}{\partial t}+\dfrac{\partial p}{\partial r}+\mu(r,t)p=f & \text{in } Q=\Omega\times(0,T), \\ p(0,t)=v(t) & \text{in } (0,T), \\ p(r,0)=u(r) & \text{in } \Omega=(0,r_m), \end{cases} \qquad (1)$$

where t denotes time, r denotes age, r_m is the highest age ever attained by individual of the population, $p(r,t)=\dfrac{\partial N(r,t)}{\partial r}$ is called age density of the population, $N(r,t)$ denotes the amount of population aged less than r at time t, $\mu(r,t)$ is the relative mortatity, $f(r,t)$ is the age density of migrants, $v(t)$ is the absolute infant fertility rate of population, $u(r)$ is an initial age density of the population.

On the basis of the paper [1], we have obtained the following results:

Theorem 1 Let $\mu(r,t)\in\mathscr{D}(\bar{Q})$; let $s\geqslant 1$ with $s\neq$ integer $+\frac{1}{2}$. Let f,v and u be given with $f\in\Xi^{-s}(Q)$, $v\in\Xi^{-(s-\frac{1}{2})}(0,T)$ and $u\in\Xi^{-(s-\frac{1}{2})}(\Omega)$. There exists an unique distribution $p(r,t)$ in $\mathscr{H}_{-(s-1)}(Q)$ such that

$$\langle p,\mathscr{P}^*q\rangle=\langle f,\bar{q}\rangle+\langle v,\overline{q(0,t)}\rangle+\langle u,\overline{q(r,0)}\rangle \quad \forall q\in X^s(Q).$$

Furthermore $\{f,v,u\}\longrightarrow p$ is a continuous linear mapping of

$$\Xi^{-s}(Q)\times\Xi^{-(s-\frac{1}{2})}(0,T)\times\Xi^{-(s-\frac{1}{2})}(\Omega)\longrightarrow\mathscr{H}_{-(s-1)}(Q).$$

The distribution p is a weak solution of the problem (1), where

$$\mathscr{P}^*=\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial r}+\mu(r,t)\right)^*=-\frac{\partial}{\partial t}-\frac{\partial}{\partial r}+\mu(r,t);$$

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$$\mathscr{H}_{-(s-1)}(Q)=\{p\,|\,p\in H^{-(s-1)}(Q),\ \mathscr{P}p\in\Xi^{-s}(Q)\},$$

provided with the norm of the graph

$$\|p\|_{\mathscr{H}_{-(s-1)}(Q)}=(\|p\|^2_{H^{-(s-1)}(Q)}+\|\mathscr{P}p\|^2_{\Xi^{-s}(Q)})^{1/2},$$

it is a Hilbert space; the definitions of $\Xi^{-s}(\Omega)$, $\Xi^{-s}(0,T)$ and $\Xi^{-s}(Q)$ are analogous to [2];

$$X^s(Q)=\{q\,|\,q\in H^s(Q),\ q(r_m,t)=q(r,0)=0,\ \mathscr{P}^*q\in H_0^{s-1}(Q)\},$$

Theorem 2 Let μ, v, u and f be given and satisfy the conditions of Theorem 1, where $s\geq 0$, $s=\text{integer}+\frac{1}{2}$. Then the problem (1) is well-posed in space

$$\mathring{\mathscr{H}}_{-(s-1)}(Q)=\{p\,|\,p\in(H_{00}^{s-1}(Q))',\mathscr{P}p\in\Xi^{-s}(Q)\}.$$

Corllary 1 Assume that the hypothese of Theorem 1 are satisfied, where $s=2$ and $u=\delta(r-r_0)$. Then the problem (1) admits a unique weak solution p in $\mathscr{H}_{-1}(Q)$ $\subset H^{-1}(Q)$. Furthermore we have:

$$p(r,t)=\begin{cases} e^{-\int_{r-t}^{r}\mu(\rho,\rho+t-r)d\rho}[\delta(r-r_0-t)+\int_0^t f(x+r-t,x)e^{\int_0^x\mu(\tau+r-t,\tau)d\tau}dx], & \text{if } t<r,\\ e^{-\int_0^r\mu(\rho,\rho+t-r)d\rho}[v(t-r)+\int_0^r f(y,y+t-r)e^{\int_0^y\mu(\rho,\rho+t-r)d\rho}dy] & \text{if } t>r. \end{cases} \quad (2)$$

From Formula (2), we may deduce:

Theorem 3 Let $\bar{S}_0$ denote the practicable average life span of individuals, $\bar{S}_{r_0}$ denote the residual practicable average life span of individuals aged r_0. Then $\bar{S}_0$ and $\bar{S}_{r_0}$, respectively, can be expressed by

$$\bar{S}_0=\int_0^{r_m}e^{-\int_0^t\mu(x,x)dx}dt \text{ and } \bar{S}_{r_0}=\int_{r_0}^{r_m}e^{-\int_{r_0}^r\mu(x,x-r_0)dx}dr,$$

or

$$\bar{S}_0=\sum_{i=0}^{r_m}e^{-\sum_{k=0}^{i}\bar{\mu}_k} \quad\text{and}\quad \bar{S}_{r_0}=\sum_{i=r_0}^{r_m}e^{-\sum_{k=r_0}^{i}\bar{\mu}'_k},$$

where

$$\bar{\mu}_k=\int_k^{k+1}\mu(x,x)dx \text{ and } \bar{\mu}'_k=\int_k^{k+1}\mu(x,x-r_0)dx.$$

References

[1] Chen Renzhao, On Stability of Non-Stationary Population Control Systems and Critical Fertility Rates of Women, Paper at the 3rd IFAC Workshop, June 23-24, 1982, Rocquencourt, France.

[2] Lions, J. L. & Magenes, E., Non-Homogeneous Boundary Value Problems and Applications, II, Springer-Verlag, Berlin, 1972.