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# Structure and Cardinality of the Class $\mathfrak{A}(R,S)$ of (0,1)-Matrices*

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In this paper we present the concept of minimally majorized corresponding segment and of decomposition sequences for a pair of majorized vectors. A detailed analysis of the structure of the total chain, and a systematic discussion on the structure and cardinality of $\mathfrak{A}(R,S)$ have been given. Moreover, the lower bound given by Wei Wandi[1] has been improved. In order to save space the reader is supposed to be familiar with the papers [1]—[4], so that terms, symbols and some results in these papers will be used without explanation. All vectors mentioned in this paper are those with non-negative integers as components, and all matrices with zeros and ones as elements.

**Definition 1** A maximal matrix is called a maximal upper matrix whenever its transpose is a maximal one (see [4]; p62).

**Definition 2** Let $S=(s_1,s_2,\cdots,s_n)$ be a vector. Then

$$S[i,j]=(s_i,s_{i+1},\cdots,s_j) \qquad (1\leqslant i\leqslant j\leqslant n)$$

is called $[i,j]$—segment of $S$, and $[i,j]$ and $j-i+1$ are called the interval subscripts and the length of $S[i,j]$, respectively. Definitions for $S[i,j)$, $S(i,j]$ and $S(i,j)$ may be given similarly. Let $A$ be a $(0,1)$-matrix of size $m$ by $n$. $A[i,j]$ is used to denote a submatrix consisting of $i$th to $j$th column of $A$. Similarly, $A(i,j]$, $A[i,j)$ and $A(i,j)$ will be used to denote other three sorts of submatrices of $A$.

**Definition 3** Let $S$ and $S^*$ be two vectors with $S\prec S^*$. Let $\hat{S}$ and $\hat{S}^*$ be two vectors of which the components of $S$ and $S^*$ are respectively listed in non-increasing order. If $\hat{S}[i,j]\prec\hat{S}^*[i,j]$, then the $\hat{S}[i,j]$ and $\hat{S}^*[i,j]$ are called a pair of majorized corresponding segment of $S$ and $S^*$. Besides, if $\hat{S}[i,k]\nprec\hat{S}^*[i,k]$ for all $k\in[i,j)$, then $\hat{S}[i,j]$ and $\hat{S}^*[i,j]$ are called a pair of minimally majorized corresponding segment of $S$ and $S^*$. If $\hat{S}(i_{j-1},i_j]$ and $\hat{S}^*(i_{j-1},i_j]$ $(1\leqslant j\leqslant k)$ are minimally majorized corresponding

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segments of $S$ and $S^*$, where $\{i_j\}_{0<j<k}$ is a strictly increasing integer sequence with $i_0=0$ and $i_k=n$, then $\{\hat{S}(i_{j-1},i_j]\}_{1<j<k}$ and $\{\hat{S}^*(i_{j-1},i_j]\}_{1<j<k}$ are called decomposition sequences for $S$ and $S^*$, $k$ is called the length of the sequences.

Obviously, if the order of equal components of $S$ and of $S^*$ are not considered, then decomposition sequences for $S$ and $S^*$ are unique.

**Theorem 1** Let $R=(r_1,r_2,\cdots,r_m)$ and $S=(s_1,s_2,\cdots,s_n)$ be two non-increasing vectors. Let $\bar{A}$ be the maximal upper matrix with row sum vector $R$ and column sum vector $\bar{S}$. Suppose that $S\prec\bar{S}$, $\{S(i_{j-1},i_j]\}_{1<j<k}$ and $\{\bar{S}(i_{j-1},i_j]\}_{1<j<k}(i_0=0,i_k=n)$ are decomposition sequences for $S$ and $\bar{S}$. Let $S(i_{j-1},i_j]$ and $\bar{S}(i_{j-1},i_j]$ be denoted by $S_j$ and $\bar{S}_j$ respectively, and let the row sum vector of $\bar{A}(i_{j-1},i_j]$ be denoted by

$$R_j=(r_{j1},r_{j2},\cdots r_{jn}),1\leqslant j\leqslant k.$$

Then

$$\mathfrak{A}(R,S)=\{A=(A(i_{j-1},i_j])_{1<j<k}\,|\,A(i_{j-1},i_j]\in\mathfrak{A}(R_j,S_j),1\leqslant j\leqslant k\},\tag{1}$$

$$|\mathfrak{A}(R,S)|=\prod_{j=1}^{k}|\mathfrak{A}(R_j,S_j)|.\tag{2}$$

Let

$$A(i_{j-1},i_j]=\begin{pmatrix}A_{1j}\\A_{2j}\\A_{3j}\end{pmatrix}\begin{matrix}\bar{s}_u\\\bar{s}_{v+1}-\bar{s}_u\\m-\bar{s}_{v+1}\end{matrix}\qquad u=i_j,v=i_{j-1}.$$

Then every element in $A_{1j}$ is an invariant 1 of $A$, every element in $A_{3j}$ is an invariant 0 of $A$ (on the definition of invariant 1 see [4], and an analogous definition holds for an invariant 0), no one of elements in $A_{2j}$ is an invariant 1 and is an invariant 0.

The theroem is mainly proved by Ryser's interchange theroem[4]. It settles the distribution and enumeration problem for the invariant 1 and the invariant 0 in $\mathfrak{A}(R,S)$.

**Corollary** The class $\mathfrak{A}(R,S)$ is with invariant 1's if and only if $r_1\geqslant i_1$.

Let $S'=(s'_1,\cdots,s'_n)$ and $S''=(s''_1,\cdots,s''_n)$ be two non-increasing vectors with $S''\prec S'$. Let $\{S''(i_{j-1},i_j]\}_{1<j<k}$ and $\{S'(i_{j-1},i_j]\}_{1<j<k}$ be decomposition sequences for $S''$ and $S'$. For simplicity and convenience, we assume that no corresponding segment is equal in decomposition sequences for $S''$ and $S'$. Then we have

**Theorem 2** There exists a unique integer sequence $\{t_j\}_{0<j<k}$ satisfying $0=t_0<t_1<\cdots<t_k=t$, so that

$$S''=S^{(t_k)}\prec\cdots\prec S^{(t_1)}\prec\cdots\prec S^{(0)}=S'\tag{3}$$

is the total chain between $S'$ and $S''$. Furthermore, we have following two results:

1. From $S^{(t_{j-1})}$ to $S^{(t_j)}$, $S'(i_{j-1},i_j]=S^{(t_{j-1})}(i_{j-1},i_j]$ is changed into $S^{(t_j)}(i_{j-1},i_j]=S''(i_{j-1},i_j]$ ,at the same time components beyond $(i_{j-1},i_j]$ are not changed, $j=1,2,\cdots,k$.

2. The total chain from $S'$ to $S^{(h)}(0\leqslant h\leqslant t)$ is

$$S^{(h)}\prec S^{(h-1)}\prec\cdots\prec S^{(0)}=S', \tag{4}$$

Let $h\in(t_{f-1},t_f]$, and the changing components of $S^{(h-1)}$ be $e$th and $g$th when $S^{(h-1)}$ is changed into $S^{(h)}$, $e<g$, $e,g\in(i_{f-1},i_f]$. Then $S^{(h)}(i_{j-1}i_j]=S''(i_{j-1},i_j]$ and $S'(i_{j-1},i_j]$ $(1\leqslant j\leqslant f-1)$ are minimally majorized corresponding segments, and $S^{(h)}(g,h]=S'(g,h]$. In addition, $S^{(h-1)}(i_{f-1},e)$ and $S'(i_{f-1},e)$, $S^{(h-1)}[e,g]$ and $S'[e,g]$ are two majorized corresponding segments of $S^{(h-1)}$ and $S'$, the length of decomposition sequences for $S^{(h-1)}[e,g]$ and $S'[e,g]$ is not less than 2.

Now let intervals of subscripts of decomposition sequences for $S^{(h-1)}[e,g]$ and $S'[e,g]$ be $[\lambda_0,\lambda_1),\cdots,[\lambda_{j-1},\lambda_j)$, with $\lambda_0=e$ and $\lambda_j=g+1$. Let

$$\sigma(S'',S^{(h)},S')=\omega^{(h)}+\sum_{p\in[\lambda_1,\lambda_{j-1})}\omega_p^{(h)},$$

where

$$\omega^{(h)}=\binom{S_e^{(h-1)}-S_g^{(h-1)}}{d^{(h)}},\quad d^{(h)}=\min(S_e^{(h-1)}-S_e'',S_g''-S_g^{(h-1)}),$$

$$\omega_p^{(h)}=\sum_{\mu=1}^{d^{(h)}}\binom{S_e^{(h-1)}-S'_{\lambda_{q-1}}}{\mu}\binom{S'_{\lambda_{q-1}}-S_p''}{d^{(h)}-\mu}\sum_{\nu=0}^{\mu-1}\binom{\mu}{\nu}\binom{S_p''-S_g^{(h-1)}+d^{(h)}-\mu}{d^{(h)}-\nu}$$

$$=\binom{S_e^{(h-1)}-S_p''}{d^{(h)}}\binom{S_p''-S_g^{(h-1)}+d^{(h)}}{d^{(h)}}-\sum_{\mu=0}^{d^{(h)}}\binom{S_e^{(h-1)}-S'_{\lambda_{q-1}}}{\mu}\binom{S'_{\lambda_{q-1}}-S_p''}{d^{(h)}-\mu}\binom{S_p''-S_g^{(h-1)}+d^{(h)}-\mu}{d^{(h)}-\mu},$$

for $p\in[\lambda_{q-1},\lambda_q)$, $q=2,\cdots,j-1$. Here $\omega^{(h)}$ is $\omega(S'',S^{(h-1)})$ in paper [1].

Let $R=(r_1,r_2,\cdots,r_m)$ and $S=(s_1,s_2,\cdots,s_n)$ be two vectors, and let $\hat{R}=(\hat{r}_1,\hat{r}_2,\cdots,\hat{r}_m)$ and $\hat{S}=(\hat{s}_1,\hat{s}_2,\cdots,\hat{s}_n)$ be two vectors of which the components in $R$ and $S$ are respectively listed in non-increasing order. Let $\bar{A}$ be the maximal upper matrix with row sum vector $\hat{R}$ and column sum vector $\bar{S}$, and $\hat{S}\prec\bar{S}$, the total chain from $\bar{S}$ to $\hat{S}$ be

$$\hat{S}=S^{(t)}\prec S^{(t-1)}\prec\cdots\prec S^{(1)}\prec S^{(0)}=\bar{S}.$$

Suppose the changing components in $S^{(t-1)}$ are $e$th and $g$th when $S^{(t-1)}$ is changed into $S^{(t)}(e<g)$. By theorem 2, $S^{(t)}(g,n]=\bar{S}(g,n]$. $S^{(t-1)}[e,g]$ and $\bar{S}[e,g]$ as well as $S^{(t-1)}[1,e)$ and $\bar{S}[1,e)$ is a pair of majorized corresponding segment of $S^{(t-1)}$ and $\bar{S}$. Let intervals of subscripts of decomposition sequences for $S^{(t-1)}[e,g]$ and $\bar{S}[e,g]$ be $[\lambda_0,\lambda_1),\cdots,[\lambda_{j-1},\lambda_j)$. Then $j\geqslant 2$. Let $\hat{R}_0,\hat{R}_{j+1}$ and $\hat{R}_q$ denote row sum vectors of $\bar{A}[1,e)$, $\bar{A}(g,n]$ and $\bar{A}[\lambda_{q-1},\lambda_q)$, respectively, $1\leqslant q\leqslant j$. Clearly, $\hat{R}_0,\hat{R}_{j+1}$ and $\hat{R}_q(1\leqslant q\leqslant j)$ are non-increasing vectors such that

$$\sum_{q=0}^{j+1}\hat{R}_q=\hat{R}.$$

Now we have

**Theorem 3**

$$|\mathfrak{A}(R,S)| \geqslant \sigma(\hat{S},S^{(t)},\bar{S}) \prod_{q=0}^{j+1} |\mathfrak{A}(\hat{R}_q,\hat{S}_q)| = \sigma(\hat{S},S^{(t)},\bar{S})\,|\mathfrak{A}(R,S^{(t-1)})|,$$

where $\hat{S}_0=\hat{S}[1,e),\hat{S}_{j+1}=\hat{S}(g,n],\hat{S}_q=\hat{S}[\lambda_{q-1},\lambda_q),1\leqslant q\leqslant j.$

This theorem is mainly proved by the following lemma.

**Lemma** Let $A,B\in\mathfrak{A}(R,S)$ with $A\neq B$. Then there exist two unequal corresponding columns in $A$ and $B$ at least.

The following theorem is directly deduced from theorem 3.

**Theorem 4**

$$|\mathfrak{A}(R,S)| \geqslant \prod_{1<i<t} \sigma(\hat{S},S^{(i)},\bar{S}) \geqslant \prod_{1<i<t} \omega(\hat{S},S^{(i-1)}).$$

Obviously, the result is better than Wei's[1].

**Definition 4.** $A$ vector $\bar{S}=(\bar{s}_1,\cdots,\bar{s}_n)$ is called I-type vector if its components satisfy

$$\bar{s}_1=\bar{s}_2=\cdots=\bar{s}_{n-1}>\bar{s}_n;$$

it is called II-type vector if its components satisfy

$$\bar{s}_1>\bar{s}_2=\cdots=\bar{s}_n;$$

it is called III-type vector if all of its components are equal. In particular, a vector containing only one component is of III-type vector.

**Theorem 5** On theorem 3, let $\{\bar{S}(i_{j-1},i_j]\}_{1<j<k}$ and $\{\hat{S}(i_{j-1},i_j]\}_{1<j<k}$ be decomposition sequences for $\bar{S}$ and $\hat{S}$. We have

$$|\mathfrak{A}(R,S)| = \prod_{1<i<t} \omega^{(i)} = \prod_{1<i<t} \omega(\hat{S},S^{(i-1)}), \tag{5}$$

if $\bar{S}(i_{j-1},i_j]$ are I-type or II-type or III-type vector.

We guess it is necessary for equality (5).

In addition, we give two obvious propositions.

**Proposition 1** Let $R=(r_1,\cdots,r_m)$ and $S=(s_1,\cdots,s_n)$ be two vectors. Let $\bar{A}$ be the maximal matrix with row sum vector $R$ and column sum vector $\bar{S}$, and $S\prec\bar{S}$, Suppose $m_i\,(1\leqslant i\leqslant k)$ are positive integers such that $\sum_{i=1}^{k} m_i=m$, and

$$\bar{A}=\begin{pmatrix}\bar{A}_1\\ \vdots\\ \bar{A}_k\end{pmatrix}\begin{matrix}m_1\\ \vdots\\ m_k\end{matrix},$$

where $\bar{A}_i$ is the $m_i$ by $n$ matrix with row sum vector $R_i$ and column sum vector $\bar{S}_i=(\bar{s}_{i1},\cdots,\bar{s}_{in}),1\leqslant i\leqslant k$. Let $S_i=(s_{i1},\cdots,s_{in})\,(1\leqslant i\leqslant k)$ be vectors such that $\sum_{i=1}^{k} S_i=S$ and $S_i\prec\bar{S}_i$.

Then

$$\mathfrak{A}(R,S)=\left\{A=\begin{pmatrix}A_1\\ \vdots\\ A_k\end{pmatrix}\middle| A_i\in\mathfrak{A}(R_i,S_i),\ \sum_{i=1}^{k}S_i=S,\ S_i\prec\bar{S}_i\right\};$$

$$|\mathfrak{A}(R,S)|=\sum\prod_{i=1}^{k}|\mathfrak{A}(R_i,S_i)|,$$

where the sum extends over all $S_i\prec\bar{S}_i\,(1\leqslant i\leqslant k)$ for which $\sum\limits_{i=1}^{k}S_i=S$.

Let $B=(B_1,\cdots,B_r,B_{r+1},\cdots,B_n)$ such that $B_i\,(1\leqslant i\leqslant r)$ are $s$-dimensional column vectors of 1's and $B_j\,(r+1\leqslant j\leqslant n)$ are $s$-dimensional zero vectors. First, shift $\lambda_{1j}$ 1's from $B_1$ to 0 in $B_j$ in the rows of $B\,(r+1\leqslant j\leqslant n)$, successively; secondly, shift $\lambda_{2j}$ 1's from $B_2$ to 0 in $B_j$ in the rows of $B\,(r+1\leqslant j\leqslant n)$, successively; ……; finally, shift $\lambda_{rj}$ 1's from $B_r$ to 0 in $B_j\,(r+1\leqslant j\leqslant n)$, succesively. As a result, $B$ is changed into $B^*$. Let $\mu_{i,j}=\lambda_{j,r+i}$ and $\Lambda=(\mu_{i,j})_{(n-r)\times r}$, whose column sum vector and row sum vector are denoted by $\psi=(\lambda_1,\lambda_2,\cdots,\lambda_r)$ and $\varphi=(\lambda_{r+1},\cdots,\lambda_n)$, respectively. Naturally $\lambda_k\leqslant s$, $1\leqslant k\leqslant n$. Let the counting number of all $B^*$ be denoted by $N(\Lambda;n,s,r)$. Let

$$R=\underbrace{(r,r,\cdots,r)}_{s},\quad S^*=(s-\lambda_1,\cdots,s-\lambda_r,\lambda_{r+1},\cdots,\lambda_n).$$

Then

$$\{B^*\}=\mathfrak{A}(R,S^*),\quad N(\Lambda;n,s,r)=|\mathfrak{A}(R,S^*)|.$$

It is not difficult to calulate $N(\Lambda;n,s,r)$ for small values of $s$ or for small values of $n$ and $r$.For example, taking $n=4$, $r=2$, we have

$$N(\Lambda;4,s,2)=\binom{s}{\lambda_{13},\lambda_{14},s-\lambda_{13},s-\lambda_{14}}\sum_{i=0}^{\lambda_{14}}\binom{\lambda_{14}}{i}\binom{s-\lambda_{13}}{\lambda_{23}-i}\binom{s-\lambda_{14}-\lambda_{23}+i}{\lambda_{24}}.$$

After having introduced the above, we have

**Proposition 2** Let $R$ be an $m$-dimensional vector and $S$ an $n$-dimensional vector, and let $\hat{R}=(\hat{r}_1,\hat{r}_2,\cdots,\hat{r}_m)$ and $\hat{S}=(\hat{s}_1,\hat{s}_2,\cdots,\hat{s}_n)$ be two vectors of which the components in $R$ and $S$ are respectively listed in non-increasing order. Let $\bar{A}$ be the maximal upper matrix with row vector $\hat{R}$ and column vector $\bar{S}=(\bar{s}_1,\bar{s}_2,\cdots,\bar{s}_n)$; Suppose that $\hat{S}\prec\bar{S}$. Let $\{t_i\}_{0\leqslant i\leqslant k+1}$ be a strictly increasing integer sequence with $t_0=0$ and $t_{k+1}=n$ such that $\bar{S}(t_{i-1},t_i]\,(1\leqslant i\leqslant k+1)$ are III-type vectors and $\bar{s}_{t_i}>\bar{s}_{t_i+1}$. Let $\mu^{(i)}_{\xi\eta}=\lambda^{(i)}_{\eta,t_i+\xi}$ and $\Lambda^{(i)}=(\mu^{(i)}_{\xi\eta})_{(n-t_i)\times t_i}$, whose column sum vector and row sum vector are respectively denoted by $\psi^{(i)}=(\lambda^{(i)}_1,\lambda^{(i)}_2,\cdots,\lambda^{(i)}_{t_i})$ and $\varphi^{(i)}=(\lambda^{(i)}_{t_i+1},\cdots,\lambda^{(i)}_n)$, where $\lambda^{(i)}_l\leqslant a_i=\bar{s}_{t_i}-\bar{s}_{t_i+1}\,(1\leqslant l\leqslant n)$, $i=1,2,\cdots,k$. Then

$$|\mathfrak{A}(R,S)|=|\mathfrak{A}(\hat{R},\hat{S})|=\sum\prod_{i=1}^{k}N(\Lambda^{(i)};n,a_i,t_i),$$

where the sum extends over all $\sum\limits_{l=1}^{k}\lambda_\mu^{(l)}=\bar{s}_\mu-\hat{s}_\mu,\ 1\leqslant\mu\leqslant t_1;\ -\lambda_\mu^{(1)}+\sum\limits_{l=2}^{k}\lambda_\mu^{(l)}=\bar{s}_\mu-\hat{s}_\mu,\ t_1<\mu\leqslant t_2;\ \cdots;\ -\sum\limits_{l=1}^{k}\lambda_\mu^{(l)}=\bar{s}_\mu-\hat{s}_\mu,\ t_k<\mu\leqslant n.$

Finally, let us see two examples.

**Example 1** Let $R=(6,5,4,4,4,3,3,3,2,2,1)$ and $S=(9,8,5,5,5,5)$, find a lower bound of $|\mathfrak{A}(R,S)|$.

It is easy to calculate $\bar{S}=(11,10,8,5,2,1)$ and to test $S\prec\bar{S}$. The total chain from $\bar{S}$ to $S$ is

$$S^{(0)}=\bar{S}=(11,10,8,5,2,1),\qquad S^{(1)}=(11,10,5,5,5,1),$$

$$S^{(2)}=(11,8,5,5,5,3),\qquad S^{(3)}=S=(9,8,5,5,5,5).$$

$$\sigma(S,S^{(1)},\bar{S})=\binom{8-2}{3}+\left[\binom{8-5}{3}\binom{5-2+3}{3}-\sum_{\mu=0}^{3}\binom{8-5}{\mu}\binom{5-5}{3-\mu}\binom{5-2+3-\mu}{3-\mu}\right]=39,$$

$$\sigma(S,S^{(2)},\bar{S})=\binom{10-1}{2}+3\left[\binom{10-5}{2}\binom{5-1+2}{2}-\sum_{\mu=0}^{2}\binom{10-8}{\mu}\binom{8-5}{2-\mu}\binom{5-1+2-\mu}{2-\mu}\right]=258,$$

$$\sigma(S,S^{(3)},\bar{S})=\binom{11-3}{2}+\left[\binom{11-8}{2}\binom{8-3+2}{2}-\sum_{\mu=0}^{2}\binom{11-10}{\mu}\binom{10-8}{2-\mu}\binom{8-3+2-\mu}{2-\mu}\right]$$

$$+3\left[\binom{11-5}{2}\binom{5-3+2}{2}-\sum_{\mu=0}^{2}\binom{11-10}{\mu}\binom{10-5}{2-\mu}\binom{5-3+2-\mu}{2-\mu}\right]=103.$$

By theorem 4, we get

$$|\mathfrak{A}(R,S)|\geqslant\prod_{i=1}^{3}\sigma(S,S^{(i)},\bar{S})=39\times258\times103=1036386.$$

However we get the following by theorem in [1],

$$|\mathfrak{A}(R,S)|\geqslant\binom{6}{3}\binom{9}{2}\binom{8}{2}=20160.$$

**Example 2** Let $R=(r_1,r_2,\cdots,r_m)$, where $r_i=n\,(1\leqslant i\leqslant c),\ r_j=n-1\,(c<j\leqslant b),\ r_k=1\,(b<k\leqslant a),\ r_l=0\,(a<l\leqslant m)$. Let $S=(a-d,\underbrace{b,\cdots,b}_{n-2},c+d)$, $0\leqslant d\leqslant\min(a-b,\ b-c)$, find $|\mathfrak{A}(R,S)|$.

It is easy to get $\bar{S}=(a,b,\cdots,b,c)$ and to test $S\prec\bar{S}$. In this example,

$$A^{(1)}=(\lambda_{12}^{(1)},\lambda_{13}^{(1)},\cdots,\lambda_{1n}^{(1)})^T,\qquad A^{(2)}=(\lambda_{1n}^{(2)},\lambda_{2n}^{(2)},\cdots,\lambda_{n-1,n}^{(2)}),$$

$$N(A^{(1)};n,a-b,1)=\binom{a-b}{\lambda_{12}^{(1)},\lambda_{13}^{(1)},\cdots,\lambda_{1n}^{(1)},a-b-\sum\limits_{i=2}^{n}\lambda_{1i}^{(1)}},$$

$$N(\Lambda^{(2)};n,a-b,n-1)=\binom{b-c}{\lambda_{1n}^{(2)},\cdots,\lambda_{n-1,n}^{(2)},b-c-\sum_{i=1}^{n-1}\lambda_{in}^{(2)}},$$

by proposition 2, we get

$$|\mathfrak{A}(R,S)|=\sum N(\Lambda^{(1)};n,a-b,1)\cdot N(\Lambda^{(2)};n,a-b,n-1)$$

$$=\sum\binom{a-b}{\lambda_2,\cdots,\lambda_n,a-b-\sum_{i=2}^{n}\lambda_i}\cdot\binom{b-c}{\lambda_1,\cdots,\lambda_{n-1},b-c-\sum_{i=1}^{n-1}\lambda_i},$$

where the former sum extends over all $\lambda_{1n}^{(1)}+\sum_{i=1}^{n-1}\lambda_{in}^{(2)}=d$ for which $-\lambda_{1j}^{(1)}+\lambda_{jn}^{(2)}=0$, $2\leqslant j\leqslant n-1$, and the latter sum over all $\lambda_i\geqslant 0$ for which $\sum_{i=1}^{n}\lambda_i=d$.

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# (0·1)－矩阵类 $\mathfrak{A}(R,S)$ 的结构和基数

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## 提 要

本文提出了两个保优向量间的极小保优对应段和分解列的概念，较详细地分析了全链的结构（见定理 2），解决了 $\mathfrak{A}(R,S)$ 中恒 1 与恒 0 的分布和计数问题（见定理 1），讨论了 $\mathfrak{A}$(R,S) 的结构和基数（见定理3—5），并改进了魏万迪[1]给出的 $|\mathfrak{A}(R,S)|$ 的下界．最后，我们给出了两个命题和例子（本文基本结果曾于一九八二年四月在华中工学院数学系组合数学讨论班上报告过）．