第4卷 第2期
1984年4月

数学研究与评论
JOURNAL OF MATHEMATICAL
RESEARCH AND EXPOSITION

Vol.4 No.2
April 1984

On Near-optimal Linear Digital Tracking Filters*

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Although the Kalman filter gives an optimal recursive data processing scheme, it has certain disadvantages such as its complexity in calculations. For this reason, in real time tracking, the third order linear predictor-corrector filter, or commonly known as the α-β-γ filter, is quite often used instead. In particular, if the measurement and input noise processes are not white, it is usually necessary to sacrifice optimality for physical feasibility in the tracking process. In this paper, we discuss a near-optimal digital tracking filter in the form of an α-β-γ filter which is very efficient in applications. Near-optimality is obtained by choosing the gain matrix as the limit of the sequence of Kalman gain matrices. Input to observation noise ratios in terms of the stochastic parameters will be given as functions of α, β, and γ. This allows the user to design an α-β-γ tracking filter to attain near-optimal performance. z-transforms will be applied to uncouple the filter equations for analyzing purposes.

1 Definition and existence of a near-optimal filter

The discrete-time system considered in this paper will be the usual time-invariant state-space equations:

$$\begin{cases} \mathrm{x}_{k+1} = \Phi \mathrm{x}_k + \mathrm{u}_k \\ \mathrm{z}_k = H^T \mathrm{x}_k + \mathrm{v}_k, \end{cases}$$

where x_k and z_k are the state and observation vectors, respectively, at the k^{th} instance, Φ the state transition matrix, H^T the transpose of the measurement matrix, and

(1) $$\mathrm{u}_{k+1} = G\mathrm{u}_k + \xi_k,$$

(2) $$\mathrm{v}_{k+1} = F\mathrm{v}_k + \eta_k,$$

the dynamic and observation noises, respectively, with

$$\mathrm{u}_0 \sim N(0, \sigma^2) \text{ and } \mathrm{v}_0 \sim N(0, \tau^2).$$

*Received Nov. 15, 1983.

Of course, if white noise input is considered, then both G and F are zero matrices. More generally, we will consider colored noise processes with

(3) $$E\xi_i\xi_j^T = Q\delta_{ij},\ E\eta_i\eta_j^T = R\delta_{ij},\ \text{and}\ E\xi_i\eta_j^T = 0.$$

As usual, we also assume that $E\mathrm{x}_0 = \bar{\mathrm{x}}_0$, Var $\mathrm{x}_0 = P_0$, $E\mathrm{x}_0\mathrm{u}_0^T = 0$, $E\mathrm{x}_0\mathrm{v}_0^T = 0$, $E\mathrm{x}_0\xi_k^T = 0$, $E\mathrm{x}_0\eta_k^T = 0$, $E\mathrm{u}_0\xi_k^T = 0$, $E\mathrm{v}_0\eta_k^T = 0$, $E\mathrm{u}_0\eta_k^T = 0$, $E\mathrm{v}_0\xi_k^T = 0$, and $E\mathrm{u}_0\mathrm{v}_0^T = 0$.

We now consider the Kalman filter equations:

$$\mathrm{y}_{k+1/k} = (\Phi - K_kH^T)\mathrm{y}_{k/k-1} + K_k\mathrm{z}_k$$

with $\mathrm{y}_{0/-1} = \bar{\mathrm{x}}_0$ where $\mathrm{y}_{k/k-1} = E[\mathrm{x}_k|\mathrm{z}_0, \cdots, \mathrm{z}_{k-1}]$ and K_k is the Kalman gain matrix at the k^{th} instance to be described below. These Kalman equations are valid for white noise processes. They are also valid for colored noise input processes provided that the inverse matrix in the Riccati equation to be discussed below exists and that Φ and H respectively are replaced by

$$\begin{pmatrix} \Phi & I & 0 \\ 0 & G & 0 \\ 0 & 0 & F \end{pmatrix} \text{ and } \begin{pmatrix} H \\ 0 \\ I \end{pmatrix}.$$

Setting $\mathrm{y}_k = \mathrm{y}_{k/k} = E[\mathrm{x}_k|\mathrm{z}_0, \cdots, \mathrm{z}_k]$, we have the well-known relationship

$$\mathrm{y}_{k+1/k} = \Phi Y_k.$$

Also, if $\sum_{k/k-1}$ denotes the error covariance matrix associated with the estimate $\mathrm{y}_{k/k-1}$, then it can be calculated recursively by using the discrete-time Riccati equation

$$\Sigma_{k+1/k} = \Phi[\Sigma_{k/k-1} - \Sigma_{k/k-1}H(H^T\Sigma_{k/k-1}H + R)^{-1}H^T\Sigma_{k/k-1}]\Phi^T + Q$$

with $\Sigma_{0/-1} = P_0$. The Kalman gain matrix at the k^{th} instance is then given by

$$K_k = \Phi\Sigma_{k/k-1}H(H^T\Sigma_{k/k-1}H + R)^{-1}.$$

As usual, we also say that the pair $[\Phi, H]$ of matrices is completely detectable if the corresponding pair $[\Phi^T, H]$ is completely stabilizable; that is, whenever $H^Tg = 0$ and $\Phi g = \lambda g$ for some constant λ, we must have $|\lambda| < 1$ or $\mathrm{g} = 0$, (cf.[1]). The following result is contained in [1,5].

LEMMA 1 *Let $[\Phi, H]$ be completely detectable and $[\Phi, S]$ be completely stabilizable for any S with $SS^T = Q$. Then for any nonnegative definite symmetric initial condition Σ_{k_0/k_0-1}, the limit as $k\to\infty$ of $\Sigma_{k+1/k}$ exists. Furthermore, this limit Σ is independent of Σ_{k_0/k_0-1} and satisfies the steady-state discrete-time Riccati equation*

$$\Sigma = \Phi[\Sigma - \Sigma H(H^T\Sigma H + R)^{-1}H^T\Sigma]\Phi^T + Q.$$

Hence, for any nonnegative definite initial condition Σ_{k_0/k_0-1}, the sequence of Kalman gains K_k, $k = k_0, k_0+1, \cdots$, converges to some matrix K defined by

$$K = \Phi\Sigma H(H^T\Sigma H + R)^{-1},$$

which is independent of the initial condition Σ_{k_0/k_0-1}. We call K the *limiting Kalman gain* and the corresponding constant gain filter

(4) $$\begin{cases} \mathrm{y}_{k+1/k} = (\Phi - KH^T)\mathrm{y}_{k/k-1} + K\mathrm{z}_k \\ \mathrm{y}_{0/-1} = \mathrm{x}_0 \text{ and } \mathrm{y}_{k+1/k} = \Phi\mathrm{y}_k \end{cases}$$

the associated *limiting Kalman filter*. Since the Kalman filter is an optimal recursive data processing algorithm, we will say that the limiting Kalman filter is a *near-optimal digital filter*. The idea of working with the possible limits of sequences of Kalman gains should be credited to R. E. Green and W. L. Shepherd, and the limiting Kalman filter was introduced and studied in some detail for white noise processes in [3]. Note that if the state transition matrix is nonsingular, then the near-optimal filter described here can also be expressed by

(5) $$\mathbf{y}_{k+1}=\Phi\mathbf{y}_k+(\Phi^{-1}K)(\mathbf{z}_{k+1}-H^T\Phi\mathbf{y}_k),k=-1,0,\cdots,\ \text{with}\ \mathbf{y}_{-1}=\Phi^{-1}\bar{\mathbf{x}}_0.$$

For practical purposes, we only study the important model where the state transition matrix relates the position, velocity, and acceleration components of the state vector from its k^{th} instance to its $(k+1)^{st}$ instance with sampling time $h>0$, and only position observations are given. Hence, when white and colored(nonwhite) noise inputs are considered, we have $\Phi=\Phi_w$, $H=H_w$, and $\Phi=\Phi_c$, $H=H_c$ respectively, where

$$\Phi_w=\begin{pmatrix}1 & h & h^2/2\\ 0 & 1 & h\\ 0 & 0 & 1\end{pmatrix},\ H_w=\begin{pmatrix}1\\0\\0\end{pmatrix},\qquad \Phi_c=\begin{pmatrix}\Phi_w & I & 0\\ 0 & G & 0\\ 0 & 0 & F\end{pmatrix},\ \text{and}\ H_c=\begin{pmatrix}1\\0\\0\\0\\0\\0\\1\end{pmatrix}.$$

Here, G and F are the matrices that describe the colored noise process given in (1) and (2), and as usual, we write the covariance matrices Q and R in (3) as

$$Q=\begin{pmatrix}\sigma_p & 0 & 0\\ 0 & \sigma_v & 0\\ 0 & 0 & \sigma_a\end{pmatrix}\ \text{and}\ R=[\sigma_m],$$

where $\sigma_p\ \sigma_v\ \sigma_a\geqslant 0$ and $\sigma_m>0$ are stochastic parameters representing position, velocity, acceleration, and position measurement respectively. In addition, let $\tilde{Q}$ be a 7×7 matrix defined by

$$\widetilde{Q}=\begin{pmatrix}0 & 0 & 0\\ 0 & \tilde{Q} & 0\\ 0 & 0 & R\end{pmatrix}.$$

We have the following result.

LEMMA 2 *Suppose that the eigenvalues of the matrices G and F are of absolute value less than 1. Then the pairs $[\Phi_w, H_w]$ and $[\Phi_c,H_c]$ are both completely detectable, and a necessary and sufficient condition for the pairs $[\Phi_w, S]$ and $[\Phi_c, \tilde{S}]$ to be completely stabilizable for any S and $\tilde{S}$ satisfying $SS^T=Q$ and $\tilde{S}\tilde{S}^T=\tilde{Q}$ is that σ_a is nonzero.*

For white noise process we certainly do not have to assume anything on the (zero) matrices G and F, and as long as σ_a is nonzero, the above lemmas imply that a near-optimal constant gain digital tracking filter always exists. For colored (non-white) noise process, we have the following result.

THEOREM 1 *Let the dynamic and observation colored noises* $\mathbf{u}_k$ *and* $\mathbf{v}_k$ *in* (1) *and* (2) *be zero-mean stationary such that the eigenvalues of* G *and* F *are of absolute value less than one. Then if* $\sigma_a \neq 0$, *the discrete time system*

$$(6)\qquad \begin{cases} \mathbf{x}_{k+1} = \Phi \mathbf{x}_k + \mathbf{u}_k \\ \mathbf{z}_k = H^T \mathbf{x}_k + \mathbf{v}_k \end{cases}$$

admits a near- optimal digital tracking filter

$$\mathbf{y}_{k+1/k} = [\Phi_c - K_c H_c^T]\mathbf{y}_{k/k-1} + K\mathbf{z}_k,$$

with

$$y_{0/-1} = \begin{pmatrix} \bar{\mathbf{x}}_0 \\ 0 \\ 0 \end{pmatrix}, \mathbf{y}_{k+1/k} = \Phi_c \mathbf{y}_k,$$

where the limiting Kalman gain matrix K_c *is given by*

$$K_c = \Phi_c \Sigma_c H_c (H_c^T \Sigma_c H_c)^{-1} = \sigma_{0,c}^{-1} \Phi_c \Sigma_c H_c$$

and the limiting error convariance matrix Σ_c *satisfies the Riccati equation*

$$\Sigma_c = \Phi_c [\Sigma_c - \sigma_{0,c}^{-1} \Sigma_c L_c \Sigma_c] \Phi_c^T + \tilde{Q}$$

with

$$\Sigma_c = [\sigma_{ij}]_{7\times 7},$$

$$\sigma_{0,c} = \sigma_{11} + \sigma_{17} + \sigma_{71} + \sigma_{77}$$

and

$$L_c = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ \vdots & & & & & & \vdots \\ 0 & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}_{7\times 7}.$$

When white noise input is considered, the corresponding Riccati equation will be reduced to be

$$\Sigma_w = \Phi_w [\Sigma_w - \sigma_{0,w}^{-1} \Sigma_w L_w \Sigma_w] \Phi_w^T + Q$$

with

$$\Sigma_w = [\sigma_{ij}]_{3\times 3}, \quad \sigma_{0,w} = \sigma_{11} + \sigma_m$$

and

$$L_w = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

We remark that the assumption in Lemma 2 and the above theorem that the matrices G and F have eigenvalues with absolute value less than one is not very restrictive. In fact, we have the following

PROPOSITION 1. *If the dynamic and observation colored noises* $\mathbf{u}_k$ *and* $\mathbf{v}_k$ *in* (1) *and* (2) *are zero-mean stationary with diagonal matrices* G *and* F, *then the eigenvalues (or diagonal entries) of* G *and* F *are all of absolute value less than one.*

2 On near-optimal α-β-γ filters

We now introduce the notion of near-optimality of an α-β-γ filter. An α-β-γ filter will be said to be near-optimal if

$$K_c=\begin{pmatrix}\Phi_v & I & 0\\ 0 & G & 0\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix}\begin{pmatrix}\alpha\\ \beta/h\\ \gamma/h^2\end{pmatrix}\\ U\\ \theta\end{pmatrix},$$

where U and θ are stochastic parameters. When the colored noise input processes become white, an α-β-γ filter is near-optimal if

$$K_w=\Phi_w\begin{pmatrix}\alpha\\ \beta/h\\ \gamma/h^2\end{pmatrix}.$$

Since K_c always exists whenever $\sigma_a\neq 0$, provided that the eigenvalues of G and F are of absolute value less than one, and depends on the stochastic parameters σ_p, σ_v, σ_a, and σ_m, we can obtain a near- optimal α-β-γ filter by choosing the values of α, β, γ, θ and the vector U as functions of these parameters. We will see that the dependence is only on the ratios σ_p/σ_m, σ_v/σ_m and σ_a/σ_m.

The most important case in colored noise processes is $F\neq 0$. For convenience, we only consider the case when $G=0$ here. we have the following result.

THEOREM 2 *Let $G=0$, $F=[r]\neq 0$ and $\sigma_a\neq 0$. Then the zero-mean stationary colored noise α-β-γ digital filter is near-optimal if and only if the following conditions are satisfied:*

(i) $$(2\alpha+2\beta+\gamma)\gamma>0,$$

(ii) $$(r-1)(r-1-r\theta)\alpha+r\theta\beta-\frac{r(r+1)}{2(r-1)}\theta\gamma>0,$$

(iii) $$(4\alpha+\beta)(\beta+\gamma)+3(\beta^2-2\alpha\gamma)-4\frac{(r-1-\theta\gamma)}{r-1}\gamma\geqslant 0,$$

(iv) $$(\beta+\gamma)\gamma\geqslant 0,$$

(v) $$(r+1)(r\theta+r-1)\alpha+\frac{r(r+1)}{r-1}\theta\beta+\frac{r(r+1)^2}{2(r-1)^2}\gamma-r^2+1\geqslant 0,$$

(vi) $$\frac{\sigma_v}{\sigma_a}=\frac{h^2}{\gamma^2}(\beta^2-2\alpha\gamma),$$

(vii) $$\frac{\sigma_p}{\sigma_a}=\frac{h^4}{2\gamma^2}\left[(2\alpha+2\beta+\gamma)\alpha-4\,\frac{r-1-r\theta}{r-1}\beta-\frac{4r\theta}{(r-1)^2}\gamma\right],$$

(viii) $$\frac{\sigma_m}{\sigma_a}=\frac{h^4}{\gamma^2}\left[(r+1)(r\theta+r-1)\alpha+\frac{r(r+1)}{r-1}\theta\beta+\frac{r(r+1)^2}{2(r-1)^2}\gamma+r^2\theta^2-r^2+1\right],$$ *and*

(ix) *the matrix*

$$\tilde{\Sigma}=[\tilde{\sigma}_{ij}]_{4\times 4}$$

is a nonnegative definite symmetric matrix, with

$$\tilde{\sigma}_{11}=\frac{1}{r-1}\left[(r-1-\theta r)\,\alpha+\frac{r}{r-1}\theta\beta-\frac{r(r+1)}{2(r-1)^2}\theta\gamma\right],$$

$$\tilde{\sigma}_{12}=\frac{1}{r-1}\left[(r-1-\theta r)\,\beta+\frac{\theta r}{r-1}\gamma\right],$$

$$\tilde{\sigma}_{13}=\frac{1}{r-1}(r-1-\theta r)\gamma,$$

$$\tilde{\sigma}_{14}=\frac{r\theta}{r-1}\left(\alpha-\frac{\beta}{r-1}+\frac{r+1}{2(r-1)^2}\gamma\right),$$

$$\tilde{\sigma}_{22}=\frac{1}{4}(4\alpha+\beta)\ (\beta+\gamma)+\frac{3}{4}(\beta^2-2\alpha\gamma)-\frac{(r-1-\theta r)}{r-1}\gamma,$$

$$\tilde{\sigma}_{23}=\left(\alpha+\frac{1}{2}\beta\right)\gamma,$$

$$\tilde{\sigma}_{24}=\frac{r\theta}{r-1}\left(\beta-\frac{\gamma}{r-1}\right),$$

$$\tilde{\sigma}_{33}=(\beta+\gamma)\gamma,$$

$$\tilde{\sigma}_{34}=\frac{r\theta}{r-1}\gamma,\ and$$

$$\tilde{\sigma}_{44}=\frac{1}{1-r^2}\left[(r+1)\ (r\theta+r-1)\alpha+\frac{r(r+1)}{r-1}\theta\beta+\frac{r(r+1)^2}{2(r-1)^2}\gamma-r^2+1\right].$$

Here, conditions (i) and (ii) are used to guarantee the existence of $\sigma_{0,c}^{-1}$ and a non-zero Σ_c. When $G\neq 0$, the method to obtain the above theorem also applies. Since its statement is fairly complicated, we do not include it in this paper. On the other hand, when both G and F are zero; that is, when white noise input is considered, the above theorem reduces to the following result obtained in [3].

COROLLARY *Let $\sigma_a\neq 0$ and $\sigma_m\neq 0$. Then the α-β-γ digital filter for white noise input processes is near-optimal if and only if the following conditions are satisfied:*

(i) $$0<\alpha<1,\gamma>0,$$

(ii) $$\sqrt{2\alpha\gamma}\leqslant\beta\leqslant\frac{\alpha}{2-\alpha}\left(\alpha+\frac{\gamma}{2}\right),\ aud$$

(iii) *the matrix*

$$\bar{p}=\begin{pmatrix}\alpha & \beta & \gamma\\ \beta & \beta\left(\alpha+\beta+\frac{\gamma}{4}\right)-\frac{\gamma}{2}(2+\alpha) & \gamma\left(\alpha+\frac{\beta}{2}\right)\\ \gamma & \gamma\left(\alpha+\frac{\beta}{2}\right) & \gamma(\beta+\gamma)\end{pmatrix}$$

is nonnegative definite. Furthermore, if the conditions (i), (ii), (iii) *are satisfied, then the stochastic parameters σ_p, σ_v, σ_a, σ_m that guarantees near-optimality satisfy:*

(iv) $$\frac{\sigma_p}{\sigma_m}=\frac{1}{1-\alpha}\left(\alpha^2+\alpha\beta+\frac{1}{2}\alpha\gamma-2\beta\right),$$

(v) $$\frac{\sigma_v}{\rho_m}=\frac{1}{1-\alpha}\left(\beta^2-2\alpha\gamma\right)h^{-2},\ and$$

(vi) $$\frac{\sigma_a}{\sigma_m}=\frac{1}{1-\alpha}\gamma^2h^{-4}.$$

3 Analysis of α-β-γ-θ filters

We again restrict ourselves to the important special case when $G=0$. Hence, a near-optimal α-β-γ filter for colored noise input processes is characterized by the equation

$$K_c=\begin{pmatrix}1 & h & h^2/2 & 0\\ 0 & 1 & h & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & r\end{pmatrix}\begin{pmatrix}\alpha\\ \beta/h\\ \gamma/h^2\\ \theta\end{pmatrix}.$$

Since the extra stochastic parameter θ can also be controlled according to the measurement noise input, we will call this an α-β-γ-θ digital tracking filter. The filter equation can be written as

(7)
$$\mathbf{y}_{k+1}=A\mathbf{y}_k+\mathbf{z}_{k+1}\mathbf{w},$$

where

$$\begin{bmatrix}1-\alpha & (1-\alpha)h & \frac{1}{2}(1-\alpha)h^2 & -\alpha r\\ -\beta/h & 1-\beta & \left(1-\frac{1}{2}\beta\right)h & -\beta r/h\\ -\gamma/h^2 & -\gamma/h & 1-\frac{1}{2}\gamma & -\gamma r/h^2\\ -\theta & -\theta h & -\frac{1}{2}\theta h^2 & (1-\theta)r\end{bmatrix}$$

and $\mathbf{w}=[\alpha,\beta/h,\gamma/h^2,\theta]^T$ and $\mathbf{y}_k=[y_k,\dot{y}_k,\ddot{y}_k,v_k]^T$. We now utilize the z-transform method to analyse this α-β-γ-θ filter equation. For convenience, we set $\mathbf{z}_k=0$ for $k<0$ and $y_k=\dot{y}_k=\ddot{y}_k=v_k=0$ for $k<-1$. Then if Z,Y_1,Y_2,Y_3 and V denote the z-transforms of $\{\mathbf{z}_k\}$, $\{y_k\}$, $\{\dot{y}_k\}$, $\{\ddot{y}_k\}$ and $\{v_k\}$ respectively, the filter equation (7) may be written in the z-domain as

(8)
$$[A-zI]\begin{pmatrix}Y_1\\ Y_2\\ Y_3\\ Y_4\end{pmatrix}=-zZ\mathbf{w}.$$

Hence, if the inverse z-transforms are applied, it is possible to uncouple filter equation (7) to study the position, velocity, acceleration, and noise estimate components separately. More specifically, we have the following result.

THEOREM 3 *The α-β-γ-θ digital tracking filter can be written as four uncoupled recursive digital filters:*

(i)
$$y_k=a_1y_{k-1}+a_2y_{k-2}+a_3y_{k-3}+a_4y_{k-4}+\alpha z_k+\left(-2\alpha-r\alpha+\beta+\frac{\gamma}{2}\right)z_{k-1}+\left[\alpha-\beta+\frac{\gamma}{2}+\left(2\alpha-\beta-\frac{\gamma}{2}\right)r\right]z_{k-2}-\left(\alpha-\beta+\frac{\gamma}{2}\right)rz_{k-3},$$

(ii)
$$\dot{y}_k=a_1\dot{y}_{k-1}+a_2\dot{y}_{k-2}+a_3\dot{y}_{k-3}+a_4\dot{y}_{k-4}+\frac{1}{h}\Big\{\beta z_k-[(2+r)\beta-\gamma]z_{k-1}+[\beta-\gamma+(2\beta-\gamma)r]z_{k-2}-(\beta-\gamma)rz_{k-3}\Big\},$$

(iii) $$\hat{y}_k = a_1\hat{y}_{k-1} + a_2\hat{y}_{k-2} + a_3\hat{y}_{k-3} + a_4\hat{y}_{k-4} + \frac{\gamma}{h^2}[z_k - (2+\gamma)z_{k-1} + (1+2r)z_{k-2} - rz_{k-3}],$$

(iv) $$v_k = a_1v_{k-1} + a_2v_{k-2} + a_3v_{k-3} + a_4v_{k-4} + \theta[z_k - 3z_{k-1} + 3z_{k-2} - z_{k-3}]$$

with the initial conditions y_{-1}, $\dot{y}_{-1}$, $\hat{y}_{-1}$, and v_{-1}, where

$$a_1 = -\alpha - \beta - \frac{1}{2}\gamma + (\theta - 1)r + 3,$$

$$a_2 = 2\alpha + \beta - \frac{1}{2}\gamma + \left(\alpha + \beta + \frac{1}{2}\gamma + 3\theta - 3\right)r - 3,$$

$$a_3 = -\alpha + \left(-2\alpha - \beta + \frac{1}{2}\gamma - 3\theta + 3\right)r + 1,$$

$$a_4 = (\alpha + \theta - 1)r.$$

Here, as usual, $z_k = 0$ for $k < 0$ *and* $y_1 = \dot{y}_1 = \hat{y}_k = v_k = 0$ for $k < -1$.

Another advantage of studying the α-β-γ-θ digital filter in the z-domain is that stability conditions in terms of the eigenvalues of A can be determined fairly easily. We remark that Theorem 3 here includes the analogous white noise result obtained in [3] and that the method works regardless of the near-optimality of the digital filter.

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近最优轨道线性数字滤波器

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为了克服Kalman最优线性滤波计算过于复杂的缺点，本文引进“近最优滤波”作为定常离散时间系统中大样本数据过程的一种极限处理，通过使用极限式Kalman增益矩阵，获得一种结构简单使用方便的极限式Kalman最优线性滤波。本文用扩大维数的办法讨论一般的有色噪声过程而把白噪声过程作为特殊情况导出。在近最优 α-β-γ 轨道滤波器中，我们可以通过对参数的适当选择而克服误差协方差矩阵退化的现象并且由于整个滤波器的结构简单而不致于增大计算量，从而为有色噪声数据过程的线性滤波提供一种可行的途径。在本文中，对有色噪声情形我们引进相应的 α-β-γ-θ 轨道滤波器，把随机参数比表达为 α、β、γ 和 θ 的函数，并用 z-变换进行滤波稳定性分析及滤波器的解耦，以方便使用者的设计和应用。