On the Best Degree of Approximation by Euler (E,q)-Means*

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Let $f(x)\in L_{2\pi}$ and its Fourier series by

$$f(x)\sim\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos nx+b_n\sin nx\equiv\sum_{n=0}^{\infty}A_n(x).$$

Denote by $S_n(f,x)$ its partial sums and by $E_n^q(f,x)$ its Euler (E,q)-means, i. e.

$$E_n^q(f,x)=\frac{1}{(1+q)^n}\sum_{m=0}^{n}\binom{n}{m}q^{n-m}S_m(f,x),$$

with $q\geqslant 0$ $(E_n^0\equiv S_n)$. In [1] Holland and Sahney proved the following theorem.

THEOREM A If $\omega(f,t)$ is the modulus of continuity of $f\in C_{2\pi}$, then the degree of approximation of f by the (E,q)-means of f is givens by

$$e_n(f)=\max_x|E_n^q(f,x)-f(x)|=O\left(\sum_{k=1}^{n}\frac{\omega\left(f,\frac{1}{k}\right)}{k}\right).$$

This result is not very good, because the inequality (1) give us at most the estimation $O(1)$ for any $f\doteqdot$const. Recentty we obtained the best degree of approximation of $f\in C_{2\pi}$ by $E_n^q(f,x)$.

Suppose $\omega(t)$ is modulus of continuity of some period continuous function and denote $H[\omega]=\{f\in C_{2\pi}:\omega(f,t)\leqslant\omega(t)\}$. Then the following theorem is established

THEOREM For any $q\geqslant o$ holds the estimation

$$\sup_{f\in H[\omega]}e_n(f)\sim\omega\left(\frac{1}{n}\right)\ln n\ \ (n\to\infty.$$

References

A. S. B. Holland and B. N. Sahney, On degree of approximation by Euler (E,q)-means, Studia Sci. Math. Hung. 11 (1976) 431-435.

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