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Integral Formulas for Submanifolds in Euclidean Space and Their Applications to Uniqueness Theorem*

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In present paper we derived some integral formulas for two n-dimensional submanifolds in Euclidean Space of dimension $n+p$ E^{n+p}, by which we get some generalizations of the Christoffel theorem and the Hilbert-Liebmann theorem.

§1 A generalization of the Christoffel theorem

Let M, M' be two n-dimensional compact submanifolds (without boundary) in E^{n+p}, f: $M \to M'$ a diffeomorphism such that M and M' have parallel tangent spaces and parallel normal spaces at x and $x'=f(x)$ respectively, that is M and M' have same Gauss image. Suppose that there exists a unit normal vector field e_{n+p} over M (it is also the unit normal vector field over M') such that the second fundamental form (h_{ij}) of M (resp. (h'_{ij}) of M') at e_{n+p} is positive definite symmetric matrix. Denote the mean curvature vector of M (resp. of M') by H (resp. H'), and let

$$K=\det(h_{ij}), \quad K'=\det(h'_{ij}).$$

We proved following

Theorem 1 M and M' differ by a translation if $K=K'$ and $H=H'$.

Theorem 2 Suppose that $\dim(M)=\dim(M')=2$, if $\dfrac{H}{K}=\dfrac{H'}{K'}$, then M and M' differ by a translation.

Theorem 2 is a generalization of the Christoffel theorem.

§2 Some generalizations of the Hillbert-Liebmann theorem

Let M be a compact, connected, n-dimensional submanifold in E^{n+p}.

Assumption A There exist p unit normal vector fields $e_{n+1}, e_{n+2}, \cdots, e_{n+p}$ over M such that M is umbilical with respect to e_τ $(\tau=n+1,\cdots,n+p-1)$, and M is convex with respect to e_{n+p}, that is for each $x\in M$, M is contained in one of the closed

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half-spaces

$$H_x^+ = \{y \in E^{n+p}: (y-x)\cdot e_{n+p}(x) \geqslant 0\}$$

or

$$H_x^- = \{y \in E^{n+p}: (y-x)\cdot e_{n+p}(x) \leqslant 0\}$$

and

$$M \cap \{y \in E^{n+p}: (y-x)\cdot e_{n+p}(x) = 0\} = \{x\}.$$

For e = even let

$$I_e = \frac{(-1)^{e/2}(n-e)!}{2^{e/2} n!} \delta^{i_1 \cdots i_e}_{j_1 \cdots j_e} R_{i_1 i_2 j_1 j_2} \cdots R_{i_{e-1} i_e j_{e-1} j_e}$$

where R_{ijkl} is the Riemann curvature tensor of M. I_e is called the Killing invariant.

Theorem 3 If M satisfies the assumption A, and I_e = const. for a fixed even e, then M is a sphere.

Theorem 4 If M satisfies the assumption A, and there exist two evens e, τ, $2 \leqslant \tau < e \leqslant n$, such that $\frac{I_e}{I_\tau}$ = const., then M is a sphere.

For the convex hypersurface in S^{n+1} we have

Theorem 5 Let M be a closed, strictly convex hypersurface in a sphere S^{n+1}, S_r be the r-th elementary symmetric function of the principal curvatures of M. If S_r = const. for a fixed integer r, $1 \leqslant r \leqslant n$, then M is a sphere.

Theorem 6 Let M be a closed strictly convex hypersurface in S^{n+1}. If $\frac{S_r}{S_\tau}$ = const. for two integers r and τ, $1 \leqslant \tau < r \leqslant n$, then M is a sphere.

References

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