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On Boundary Value Problems with the Shift for Nonlinear Elliptic Equations of Second Order*

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Abstract

In this paper, we consider the boundary value problem with the shift for nonlinear uniformly elliptic equations of second order in a multiply connected domain. For this sake, we propose a modified boundary value problem for nonlinear elliptic systems of first order equations, and give a priori estimates of solutions for the modified boundary value problem. Afterwards we prove by using the Schauder fixedpoint theorem that this boundary value problem with some conditions has a solution. The result obtained is the generlization of the corresponding theorem on the Poincaré boundary value problem.

§1 The formulation of the boundary value problem with the shift and the corresponding modified boundary value problem.

We discuss the nonlinear uniformly elliptic equation

$$u_{z\bar z}=F(z,u,u_z,u_{zz}),\quad F=\mathrm{Re}[Qu_{zz}+A_1u_z+\varepsilon A_2u+A_3,$$
$$Q=Q(z,u,u_z,u_{zz}),\ A_j=A_j(z,u,u_z),\ j=1,2,3,\ -\infty<\varepsilon<\infty \tag{1.1}$$

in a multiply connected domain D, and suppose that Eq. (1.1) satisfies the condition C in the bounded domain D, i. e.

1) $Q(z,u,u_z,V)$, $A_j(z,u,u_z)$ $(j=1,2,3)$ are measurable in z for all continuously differentiable functions $u(z)$ and all measurable functions $V(z)$ in $D\backslash\{0\}$, and satisfy

$$\|A_j[z,u(z),u_z]\|_{L^p(\bar D)}\leqslant d<\infty,\ j=1,2,3, \tag{1.2}$$

where $d(0<d<\infty)$, $p(2<p<\infty)$ are constants.

2) $Q(z,u,u_z,V)$, $A_j(z,u,u_z)$ $(j=1,2,3)$ are continuous in $u\in R$ (the real axis) and $u_z\in E$ (the whole plane) for almost every point $z\in D$ and $V\in E$, and

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$$Q(z,u,u_z,V)=0,\ A_j(z,u,u_z)=0,\ \text{for } z\bar{\in} D. \tag{1.3}$$

3) Eq. (1.1) satisfies the uniformly elliptic condition, i.e.

$$|F(z,u,u_z,V_1)-F(z,u,u_z,V_2)|\leqslant q_0|V_1-V_2|,0\leqslant q_0<1 \tag{1.4}$$

for almost every point $z\in D$, $u\in R$ and $u_z,V_1,V_2\in E$.

Let $\Gamma_0,\Gamma_1,\cdots,\Gamma_N$ be the boundary contours of an $N+1$-connected domain D, where $\Gamma_1\cdots,\Gamma_N$ are situated inside Γ_0. In the interior of D there are some mutually exclusive contours $\gamma_1,\cdots,\gamma_n$. Let $\Gamma_*=\Gamma_0+\cdots+\Gamma_{N_0}$, $\Gamma_{**}=\Gamma_{N_0+1}+\cdots+\Gamma_N$, $\gamma_*=\gamma_1+\cdots+\gamma_{n_0}$, $\gamma_{**}=\gamma_{n_0+1}+\cdots+\gamma_n\in C^2_\mu(0<\mu<1)$ and denote $D^-=(D_1^-+\cdots+D_n^-)\cap D, D^+=D\backslash\bar{D}^-$, where D_j^- is the domain surrounded by $\gamma_j(j=1,\cdots,n)$. Without loss of generality, we assume that D is a circular domain, and Γ_0: $|z|=1$, $z=0\in D^+$. The boundary value problem with the shift for Eq. (1.1) may be formulated as follows: Find a sectionally regular solution $u^\pm(z)$ in $D^\pm$ continuously differentiable on $\bar{D}^\pm$ and satisfying the boundary conditions:

$$\left.\frac{\partial u^+}{\partial\tau}\right|_{\tau=\alpha(t)}=G_1(t)\frac{\partial u^-}{\partial t}+G_2(t)\overline{\frac{\partial u^-}{\partial t}}+\varepsilon g_1(u^+,u^-,t)+g_2(t),t\in\gamma \tag{1.5}$$

$$\left.\frac{\partial u^+}{\partial\tau}\right|_{\tau=\alpha(t)}=G_1(t)\overline{\frac{\partial u^+}{\partial t}}+G_2(t)\frac{\partial u^+}{\partial t}+\varepsilon g_1(u^+,0,t)+g_2(t),t\in\Gamma, \tag{1.6}$$

where $\alpha(t)$ maps Γ_j, γ_j topologically onto itself which are positive shifts on $\Gamma_*+\gamma_*$ and reverse shifts on $\Gamma_{**}+\gamma_{**}$, $\alpha(t)$ has the fixed points $a_j\in\Gamma_j$, $j=0,\cdots,N_6$, and $\alpha(t)$, $G_j,g_j(j=1,2)$ satisfy that

$$\left.\begin{array}{l}
G_1(t)\begin{cases}\neq 0, & t\in\gamma_*+\Gamma_*,\\ =0, & t\in\gamma_{**}+\Gamma_{**},\end{cases}\quad G_2(t)\begin{cases}=0, t\in\gamma_*+\Gamma_*,\\ \neq 0, t\in\gamma_{**}+\Gamma_{**}\end{cases}\\
C_\nu[G_j(t),\partial D^\pm]\leqslant d,\ j=1,2,\ C_\nu[g_2(t),\partial D^\pm]\leqslant d,\\
|\alpha'(t)|>d^{-1}>0,\ |g_1(U_1,V_1,t_1)-g_1(U_2,V_2,t_2)|\leqslant d|U_1-U_2|^\nu\\
\quad +d|V_1-V_2|^\nu+d|t_1-t_2|^\nu,\ t_1,t_2\in\partial D^\pm,U_j,V_j\in R,\\
\quad j=1,\ 2,\ \frac{1}{2}<\nu<1,\ \alpha[\alpha(t)]=t,\ t\in\Gamma,C_\nu^1[\alpha(t),\partial D^\pm]\leqslant d,\\
G_1(t)\overline{G_1[\alpha(t)]}=1,\ G_1(t)\overline{g_j[\alpha(t)]}+g_j(t)=0,\ t\in\Gamma_*,\\
G_2(t)G_2[\alpha(t)]=1,\ G_2(t)g_j[\alpha(t)]+g_j(t)=0,\ t\in\Gamma_{**},
\end{array}\right\} \tag{1.7}$$

where $g_1(t)=\begin{cases}g_1[u^+(t),u^-(t),t], & t\in\Gamma_*\\ g_1[u^+(t),0,t], & t\in\Gamma_{**}\end{cases}$ for any continuously differentiable functions $u^+(z)$, $u^-(z)$ on Γ. The above boundary value problem will be denoted by **Problem F**.

Let now $K_{\Gamma_j}=\frac{1}{2\pi}\Delta_{\Gamma_j}\arg G_1(t)$, $j=0,1,\cdots,N_0$, $K_{\Gamma_j}=\frac{1}{2\pi}\Delta_{\Gamma_j}\arg G_2(t)$, $j=N_0+1,\cdots,N$, $K_{\gamma_j}=\frac{1}{2\pi}\Delta_{\gamma_j}\arg G_1(t)$, $j=1,\cdots,n_0$, $K_{\gamma_j}=\frac{1}{2\pi}\Delta_{\gamma_j}\arg G_2(t)$, $j=n_0+1,\cdots,n$. Besides, denote by f the total number of the fixed points of $\alpha(t)$ with $G(t)$

$=-1$ on Γ_{**}, and

$$K=2\left(\sum_{j=1}^{n_0}K\gamma_j-\sum_{j=n_0+1}^{n}K\gamma_j\right)+\sum_{j=0}^{N_0}K_{\Gamma_j}-\sum_{j=N_0+1}^{N}K_{\Gamma_j}-f \tag{1.8}$$

is called the index of Problem F. For the sake of convenience, we assume that $K_{\Gamma_j}=0,\ j=1,\cdots,N_0$.*

When $K\leqslant 2N_0-2$, the problem F may not have any solution for Eq.(1.1). Hence we propose the corresponding modified boundary value problem, i.e. Eq.(1.1) is represented by the system of first order equations

$$\begin{aligned}w_{\bar z}=f(z,U,w,w_z),\ \ f=\mathrm{Re}[&Q(z,U,w,w_z)w_z\\&+A_1(z,U,w)w+\varepsilon A_2(z,U,w)U+A_3(z,U,w)].\end{aligned} \tag{1.9}$$

$$U_{\bar z}=\overline{w^*} \tag{1.10}$$

in which

$$w^*(z)=\begin{cases}w(z)=\Phi[\zeta(z)]e^{\varphi(z)},\ \text{for } K\geqslant -2,\\ \Phi^*[\zeta(z)]e^{\varphi(z)}+\psi(z),\ \ \Phi^*(\zeta)=\Phi(\zeta)-g(\zeta),\\ g(\zeta)=\displaystyle\sum_{j=1}^{[\frac{|K|+1}{2}]-1}C_m\zeta^{-m},\ \text{for } K<-2,\end{cases} \tag{1.11}$$

where $g(\zeta)$ is the main part of the meromorphic function $\Phi(\zeta)$ in the neighborhood of $\zeta=0$ and the boundary conditions (1.5), (1.6) are represented by

$$w^+[a(t)]=G_1(t)w^-(t)+G_2(t)\overline{w^-(t)}+\varepsilon g_1(U^+,U^-,t)+g_2(t),\ \ t\in\gamma, \tag{1.12}$$

$$w^+[a(t)]=G_1(t)\overline{w^+(t)}+G_2(t)w^+(t)+\varepsilon g_1(U^+,0,t)+g_2(t)+h(t),\ \ t\in\Gamma, \tag{1.13}$$

$$h(t)=\begin{cases}0,\ t\in\Gamma_*,\ \text{for } K>2N_0-2,\\ \left.\begin{array}{l}iG_1(t)^{\frac12}h_j,\ t\in\Gamma_j,\ 1\leqslant j\leqslant N_0-\left[\dfrac{K+1}{2}\right]\\ 0,\ t\in\Gamma_j,\ j=0,\ N-\left[\dfrac{K+1}{2}\right]+1,\cdots,N_0\end{array}\right\}0\leqslant K\leqslant 2N_0-2,\\ iG_1(t)^{\frac12}h_j,\ t\in\Gamma_j,\ 0\leqslant j\leqslant N_0,\ \text{for } K<0,\end{cases} \tag{1.14}$$

$$\begin{cases}B_jw=\mathrm{Re}G(a_j)^{\frac12}\overline{w(a_j)}=b_j,\ j=N_0-\left[\dfrac{K+1}{2}\right]+1,\cdots,N_0+I,\\ \qquad I=K+1-2\left[\dfrac{K+1}{2}\right],\ \text{for } 0\leqslant K\leqslant 2N_0-2,\\ B_jw=b_j,\ j=1,\cdots,K-N_0+1,\ \text{for } K>2N_0-2,\ |b_j|\leqslant d,\end{cases} \tag{1.15}$$

$$\begin{cases}\mathrm{Re}iU(t)=H(t),t\in\partial D^+,H(t)=\begin{cases}0,t\in\Gamma_0,\\ H_j,t\in\Gamma_j,\end{cases}\ j=1,\cdots,N',N'\leqslant N+n=N'+n'\\ \mathrm{Re}iU(t)=H(t),t\in\partial D^-,H(t)=\begin{cases}0,t\in\gamma_j,\\ H'_j,t\in\gamma'_j,\end{cases}\ j=1,\cdots,n',n'\leqslant n,\end{cases} \tag{1.16}$$

$$\begin{cases}\mathrm{Re}U(a_0)=c_0,\ \ \mathrm{Re}U(a'_j)=c_j,j=1,\cdots,n',\\ |c_j|\leqslant d,\ \ j=0,\cdots,n',\end{cases} \tag{1.17}$$

*For the general case, we can also obtain the same result by using analogous method.

where $h_j(j=0,\cdots,N_0)$, $H_j(j=1,\cdots,N')$ are all unknown real constants to be determined appropriately, and $h_0=0$ when K is the negative odd number, $a_j\in\Gamma_j$ is a fixed point of $\alpha(t)$, $j=1,\cdots,N_0$, $a_j(j=N_0+1,\cdots,K-N_0+1,\ K>2N_0-2)$ are the dis--tinct points on Γ_0. Besides, when $K<0$ we permit that the solution of Eq.(1.9) possesses the poles of order $\leqslant\left[\frac{|K|+1}{2}\right]-1$ at $z=0$. The above modified boundary value problem with boundary conditions (1.12)—(1.17) for the system (1.9)—(1.11) will be called the problem G, and Problem G with $A_3=0$, $g_j=0$ $(j=1,2)$ will be denoted by problem G_0.

§2 A priori estimates of solutions of the modified boundary problem for the complex equation of first order.

In this section, we shall give a priori etimations of the modified boundary problem (Problem H) for the complex equation of first order

$$\begin{cases} w_{\bar z}=F(z,w,w_z),\ F=Q_1w_z+Q_2\overline{w}_{\bar z}+A_1w+A_2\overline{w}+A_3,\\ |Q_1(z)|+|Q_2(z)|\leqslant q_0<1,\ \|A_j(z)\|_{L^p(\bar D)}\leqslant d,\ j=1,2,3,p>2\end{cases}\tag{2.1}$$

with the boundary conditions

$$\begin{cases} w^+[\alpha(t)]=G_1(t)w^-(t)+G_2(t)\overline{w^-(t)}+g_2(t),t\in\gamma,\\ w^+[\alpha(t)]=G_1(t)\overline{w^+(t)}+G_2(t)w^+(t)+g_2(t)+h(t),t\in\Gamma\end{cases}\tag{2.2}$$

$$\begin{cases} B_jw=\mathrm{Re}G(a_j)^{\frac12}\overline{w(a_j)}=b_j,\ j=N_0-\left[\frac{K+1}{2}\right]+1,\cdots,N_0+I,\\ 0\leqslant K\leqslant 2N_0-2,\ j-1,\cdots,K-N_0+1,\ K>2N_0-2,\end{cases}\tag{2.3}$$

where $G_j(t)$, $g_2(t)$, $\alpha(t)$, $h(t)$, b_j are stated as in (1.12)—(1.17), and when $K<-2$, we permit that the solution of Eq. (2.1) possesses the poles of order $\leqslant\left[\frac{|K|+1}{2}\right]-1$ at $z=0$.

Theorem 2.1 Let $w(z)=\begin{cases}w^+(z),z\in D^+\\ w^-(z),z\in D^-\end{cases}$ be a solution of Problem H for the complex equation (2.1). Then when $K\geqslant-2$, $w(z)$ satisfies the estimations

$$C_\beta[w(z),\ D^\pm]\leqslant M_1,\ \||w_{\bar z}|+|w_z|\|_{L_{p_0}(\bar D^\pm)}\leqslant M_2,\tag{2.4}$$

and when $K<-2$, $w(z)$ satisfied the estimations

$$C_\beta[W(z),D^\pm]\leqslant M_3,\ \||W_{\bar z}|+|W_z|\|_{L_{p_0}(\bar D^\pm)}\leqslant M_4,\tag{2.5}$$

where $W(z)=w(z)[\zeta(z)]^\eta$, $\zeta(z)$ is a homeomorphic solution of the corresponding Beltrami equation, $\eta=\left[\frac{|K|+1}{2}\right]-1$, $\beta=\frac{p_0-2}{p_0}$, $2<p_0<\min\left(p,\frac{1}{1-\nu}\right)$, $M_j=M_j(q_0,p_0,\nu,d,D^\pm,\alpha(t))$, $j=1,\cdots,4$.

By using above estimations and the Leray-Schauder theorem, we can prove the following result.

Theorem 2.2 Under the hypothesis of Theorem 2.1, Problem H for Eq.(2.1) is solvable.

§3 The solvability of Problem F for Eq.(1.1)

In this section, we first prove that Problem G for the system (1.9), (1.10) has a solution. Afterwards, we derive the results of solvability for Problem F of the second order equation (1.1).

Theorem 3.1 Suppose that Eq.(1.1) satisfies Condition C and the constant $|\varepsilon|$ in (1.1), (1.5) and (1.6) is sufficiently small, then Problem G for the system (1.9), (1.10) is solvable,

Proof First of all, we assume that the coefficients Q, A_j $(j=1, 2, 3)$ of the first order system (1.9), (1.10) equal zero in the neighborhood $\overline{D}-D_m$ of the boundary ∂D^*, where $D_m=\{z \mid |z-t| \geqslant \frac{1}{m},\ t \in \partial D^*,\ z \in D$, m is a positive integer$\}$, and write this system in the form

$$w_{\bar{z}}=f_m(z,U,w,w_z),\quad f_m=\operatorname{Re}[Q_m(z,U,w,w_z)w_z] + A_1^{(m)}(z,U,w)w+\varepsilon A_2^{(m)}(z,U,w)U+A_3^{(m)}(z,U,w)] \tag{3.1}$$

$$u_{\bar{z}}=\overline{w^*} \tag{3.2}$$

Let us introduce the Banach space $B=C'(\overline{D^*})\times C(\overline{D^*})$ and denote by B_M the totality of the functions: $\Omega=[U(z),W(z)]$ satisfying the inequality

$$\|\Omega\|=C'[U(z),\overline{D^*}]+C[W(z),\overline{D^*}]\leqslant M_5 \tag{3.3}$$

where M_5 is a constant to be determined appropriately. It is evident that B_M is a bounded and closed set in B. We choose arbitrarily $\Omega=[U(z),W(z)]\in B_M$ and substitute $U(z)$, $W(z)$ into suitable positions of the coefficients Q, A_j $(j=1,2,3)$, $\varepsilon A_2 U$ for Eq. (3.1) and $g_1(U^+,U^-,t)$, $g_1(U^+,0,t)$ of the boundary conditions (1.12) and (1.13). According to Theorem 2.2, the problem H of Eq. (3.1) with the boundary conditions (1.12)—(1.15) has a unique solution $w(z)$, and conseqnently the function $w^*(z)$ in (1.11) is determined. Substituting $w^*(z)$ into (3.2), and similarly to above, the boundary value problem of Eq.(3.2) with the boundary conditions (1.16),(1.17) has a unique solution $u(z)$. Let $\omega=[u(z),w^*(z)]$ and denote by $\omega=T(\Omega)$ a mapping from Ω to ω. Moreover it is not difficult to see that no matter how large the constant M_5 in (3.3) is, provided that $|\varepsilon|$ in (3.1), (1.12) and (1.13) is sufficiently small, we must have

$$\begin{cases}\|\varepsilon A_2(z,U,W)U\|_{L_p(\overline{D^*})}\leqslant d,\\ C_\nu\{\varepsilon g_1[U^+(t),U^-(t),t],\partial D^*\}\leqslant d.\end{cases} \tag{3.4}$$

By using the method in Theorem 2.1, we conclude that $w^*(z)$ and $U(z)$ satisfy the estimations

$$C_\beta[w^*(z),\overline{D^*}]\leqslant M_6,\quad \||w^*_{\bar{z}}|+|w^*_z|\|_{L_{p_0}(\overline{D^*})}\leqslant M_7 \tag{3.5}$$

$$C_\beta[u(z),\overline{D^*}]\leq M_8,\| |u_{\bar z}| + |u_z| \|_{L_{p_0}(\overline{D^*})}\leq M_9 \tag{3.6}$$

where $\beta=\dfrac{p_0-2}{p_0}$, $M_j=M_j(q_0,p_0,\nu,d,D^*,\alpha(t))$, $j=6,\cdots,9$. From (3.2) and (1.16), we have

$$\begin{cases} u_{z\bar z}=\overline{w_z^*}, \\ \operatorname{Re}[iz'(s)u_z]=-\operatorname{Re}[i\overline{z'(s)}\,\overline{w^*}],\ z\in\partial D^*, \end{cases} \tag{3.7}$$

and

$$\begin{cases} \bar u_{z\bar z}=w_{\bar z}^* \\ \operatorname{Re}[i\bar u_z]=\operatorname{Re}iw^*, z\in\partial D^* \end{cases} \tag{3.8}$$

Hence u_z, $u_{\bar z}$ and $u(z)$ satisfy the estimations

$$\begin{cases} C_\beta[u_z,\overline{D^*}]+C_\beta[u_{\bar z},\overline{D^*}]\leq M_{10}=M_{10}(q_0,p_0,\nu,d,D^*,\alpha(t)), \\ C_\beta^1[u(z),\overline{D^*}]\leq M_{11}=M_{11}(q_0,p_0,\nu,d,D^*,\alpha(t)). \end{cases} \tag{3.9}$$

Now we select M_5 in (3.3) to be equal to M_6+M_{11};

On the basis of above discussion, we know that if the constant $|\varepsilon|$ in (3.1), (1.12) and (1.13) is sufficiently small, then $\omega=T(\Omega)$ maps B_M onto itself. Noting that $w^*(z)$, $u(z)$ satisfy the estimations (3.5) and (3.9), it is obvious that $\omega=T(\Omega)$ is a compact mapping in B_M. Besides we can verify that $\omega=T(\Omega)$ is a continuous mapping in B_M. Hence by Schauder fixedpoint theorem, the mapping $\omega=T(\Omega)$ possesses a fixed point $\omega=[u(z),w^*(z)]\in B_M$ and so $[u(z),w(z)]$ is a solution of Problem G for the system (3.1) and (3.2).

Apply the above result and the principle of compactness for a sequence of bounded solutions of the system (3.1) and (3.2), we can conclude that Problem G for the system (1.9) and (1.10) has a solution $[u(z),w(z)]$.

Theorem 3.2 Under the hypothesis of Theorem 3.1.

1) When the index $K>2N_0-2$, the problem F for Eq. (1.1) has $N+n$ solvability conditions.

2) When $0\leq k\leq 2N_0-2$, the total number of the solvability conditions for Problem F is not greater than $N+N_0+n-\left[\dfrac{K+1}{2}\right]$.

3) When $K<0$, Problem F for Eq.(1.1) has $N+N_0+n-K-1$ solvability conditions.

Proof 1) When $K>2N_0-2$, on the basis of Theorem 3.1, Problem G for the system (1.9), (1.10) has a solution $[u(z),w^*(z)]=[u(z),w(z)]$. Let $u(z)$, $w(z)$ substitute the boundary conditions (1.16), (1.17), and so the constants $H_j(j=1,\cdots,N')$, $H'_j(j=1,\cdots,n')$ are determined. If the constants are equal to zero, namely $H_j=0$ $(j=1,\cdots,N')$ and $H'_j=0(j=1,\cdots,n')$. Then we have

$$\operatorname{Im}u_{z\bar z}=0,\ z\in D^*, \tag{3.10}$$

$$\mathrm{Re}\,iu(t) = -\mathrm{Im}\,u(t) = 0, \quad t \in \partial D^* \tag{3.11}$$

It is easy to see that $\mathrm{Im}\,u(z) = 0$ for $z \in D^*$, i.e. $u(z)$ is a real value function and $u_z = w(z)$.

This shows that when $K > 2N_0 - 2$, Problem F for Eq. (1.1) has $N + n$ solvability conditions.

2) When $0 \leqslant K \leqslant 2N_0 - 2$, similarly to 1), we replace the solution $[u(z), w(z)]$ $(w(z) = w^*(z))$ of Problem G for the system (1.9), (1.10) into corresponding positions of the boundary conditions (1.12)—(1.17). If the constants $h_j\left(j = 1, \cdots, N_0 - \left[\frac{K+1}{2}\right]\right)$, $H_j = 0$ $(j = 1, \cdots, N')$ and $H'_j = 0$ $(j = 1, \cdots, n')$, then the solution $u(z)$ of Problem G for the system (1.9), (1.10) is a solution of Problem F for Eq. (1.1). Hence the total number of the solvability conditions for problem F is not greater than $N + N_0 + n - \left[\frac{K+1}{2}\right]$.

3) When $K < 0$, we substitute the solution $[u(z), w(z)]$ of Problem G into the boundary conditions (1.12)—(1.17) and (1.11). If $h_j = 0$ $(j = 0, \cdots, N)$, $H_j = 0$ $(j = 1, \cdots, N')$, $H'_j = 0$ $(j = 1, \cdots, n')$ and the main part $g(\zeta) = 0$ of the function $\Phi(\zeta)$ in (1.11), i.e. $C_m = 0$ $\left(m = 1, \cdots, \left[\frac{|K|+1}{2}\right] - 1\right)$, then the function $u(z)$ is a solution of Problem F for Eq. (1.1). Thus it can be seen that Problem F for Eq. (1.1) has $N + n + 2\left(\left[\frac{|K|+1}{2}\right] - 1\right) + N_0 + |K| + 1 - 2\left[\frac{|K|+1}{2}\right] = N + N_0 + n - K - 1$ solvability conditions.

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