On the Simultaneous Approximation in Lp-norm*

Zhou Songping (周颂平)

(Hangzhou University)

Let $C^{(N)}_{[-1,1]}$ be the class of N-th continuously differentiable functions on $[-1, 1]$, denote by $L_{p[-1,1]}$ the class of L_p-integrable functions on $[-1,1]$, $E_n(f)_p$ is the best approximation of $f \in L_{p[-1,1]}$ by nth algebraic polynomials, and $C(\cdot)$ is a positive constant only depending upon the quantities in the brackets. In Theorem 1, the condition that $f(x) \in C^{(N-1)}_{[-1,1]}$, $f^{(N-1)}(x)$ is absolutely continuous, and $f^{(N)}(x) \in L_{p[-1,1]}$ for $N=0$ equals that $f(x) \in L_{p[-1,1]}$.

In this paper, we will prove that

Theorem 1 Suppose that N is a nonnegative integer. If $f(x) \in C^{(N-1)}_{[-1,1]}$, $f^{(N-1)}(x)$ is absolutely continuous, and $f^{(N)}(x) \in L_{p[-1,1]}$, then

$$\|f^{(k)} - p_n^{(k)}(f)\|_{p[-1,1]} \leqslant C(N) n^k E_{n-k}(f^{(k)}),$$

where $0 \leqslant k \leqslant N$, $n \geqslant k$, $1 \leqslant p < \infty$, and $p_n(f)$ is the nth polynomial of best approximation in L_p-norm to $f(x)$, $\|\cdot\|_{p[a,b]}$ is the L_p-norm on $[a,b]$.

Theorem 2 Let $-1 < \alpha < \beta < 1$. If $f(x)$ satisfies the conditions of Theorem 1, $p_n(f)$ is defined as above, then

$$\|f^{(k)} - p_n^{(k)}(f)\|_{p[\alpha,\beta]} \leqslant C(\alpha,\beta,N) E_{n-k}(f^{(k)}), \quad 0 \leqslant k \leqslant N, \; n \geqslant k, \; 1 \leqslant p < \infty.$$

References

[1] DeVore, R. A., Degree of approximation, in "Approximation Theory II, Proc. of Conf. Austin, Texas, 1976" (C. K. Chui, G. G. Lorentz and L. L. Schumaker Eds.), Academic Press, New York, 1976, 117—161.

[2] Hasson, M., Derivatives of the algebraic polynomials of best approximation, *J. Approx. Theory*, 29 (1980), 91—102.

[3] Leviatan, D., The behavior of the derivatives of the algebraic polynomials of best approximation, *J. Approx. Theory*, 35 (1982), 169—176.

[4] Timan, A. F., "Theory of Approximation of Functions of a Real Variable," Macmillan Co., New York, 1963.

*Received Nov. 28, 1983.