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A Property of N-ary Relation Ring*

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In paper [1], Professor Xie Bangjie discussed the ring with left zero factor ideals having the condition of rising chain. Qiu Qichang also has work on the aspect [2]. By using T-ary relation Professor Wu Xuemou investigated body-shadow relation, whole-parts relation, etc, [3]. In this paper, the conception of n-ary relation ring is introduced, and a necessary and sufficient condition of a ring R being a right Noether ring is given.

Definition 1 Let $R=\{(x_1,x_2,\cdots,x_n)\}$ and operations are defined as

$$+: a+b=(x_1+y_1,\cdots,x_n+y_n),$$

$$*: ab=(x_1y_1,\cdots,x_ny_n),$$

where $a=(x_1,\cdots,x_n)$, $b=(y_1,\cdots,y_n)$, a, $b\in R$. If R be a ring for operations $+$, $*$, then we call R a n-ary relation ring.

Definition 2 Let R be a ring, L is a right ideal of R, if S is a subset of R, and there exists not zero element in S, and $SL=0$, then L is call a right zero factor ideal of R.

Theorem If there exists at least a non-zero right zero factor in R_i for any i $(i=1,\cdots,n)$, then R is a right Noether ring iff R_i's right zero factor ideals satisfy rising chain condition for every i. Where $R\{(x_1,\cdots,x_n)\,|\,x_1\in R_1,\cdots,x_n\in R_n\}$, $R_i\,(i=1,2,\cdots,n)$ are rings.

Proof By the definition of right Noether ring the verification of necessity is simple. Now we part the proof of sufficiency to two steps.

1) By the above hypothesis we know that right zero factor ideals of R_i satisfy rising chain condition. For any rising chain of right ideals of R_i

$$Q_1^{(i)}\subseteq Q_2^{(i)}\subseteq\cdots\subseteq Q_n^{(i)}\subseteq Q_{n+1}^{(i)}\subseteq\cdots$$

let $a_i\neq 0$ is a right zero factor of R_i. We denote by $r(a_i)=\{x\,|\,x\in R_i, a_ix=0\}$. Therefore $a_iQ_j^{(i)}$, $Q_j^{(i)}\cap r(a_i)$ $(j=1,2,\cdots)$ are two right zero factor ideals, and

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$$a_iQ_1^{(i)}\subseteq a_iQ_2^{(i)}\subseteq\cdots\subseteq a_iQ_n^{(i)}\subseteq a_iQ_{n+1}^{(i)}\subseteq\cdots$$

$$Q_1^{(i)}\cap r(a_i)\subseteq Q_2^{(i)}\cap r(a_i)\subseteq\cdots\subseteq Q_n^{(i)}\cap r(a_i)\subseteq Q_{n+1}^{(i)}\cap r(a_i)\subseteq\cdots.$$

According to the above condition, there exists a positive integer m_i such that

$$\left.\begin{array}{l} a_iQ_m^{(i)}=a_iQ_{m+1}^{(i)}=\cdots \\ Q_m^{(i)}\cap r(a_i)=Q_{m+1}^{(i)}\cap r(a_i)=\cdots \end{array}\right\}. \tag{A}$$

Now we prove that for any $j\geqslant m_i$, $Q_j^{(i)}=Q_{j+1}^{(i)}$. It is sufficient to show that $Q_{j+1}^{(i)}\subseteq Q_j^{(i)}$.

For arbitrary $x_{j+1}\in Q_{j+1}^{(i)}$, by (A) we know that there exists $x_j\in Q_j^{(i)}$ such that

$$a_ix_j=a_ix_{j+1},$$

so $x_j-x_{j+1}\in r(a_i)$, and x_j, $x_{j+1}\in Q_{j+1}^{(i)}$, $x_j-x_{j+1}\in Q_j^{(i)}$, therefore $x_j-x_{j+1}\in Q_{j+1}^{(i)}\cap r(a_i)$ $=Q_j^{(i)}\cap r(a_i)$, for this reason $x_j-x_{j+1}\in Q_j^{(i)}$. $Q_{i+1}\subseteq Q_i$ consequently. Therefore $Q_j=Q_{j+1}$.

2) Let $a_i\neq 0\,(i=1,2,\cdots,n)$ are as above. Take

$$a=(a_1,a_2,\cdots,a_n).$$

For any rising chain of right ideals of R

$$Q_1\subseteq Q_2\subseteq\cdots\subseteq Q_n\subseteq Q_{n+1}\subseteq\cdots,$$

$Q_j=(Q_j^{(1)},Q_j^{(2)}\cdots,Q_j^{(n)})$, then

$$Q_1^{(i)}\subseteq Q_2^{(i)}\subseteq\cdots\subseteq Q_n^{(i)}\subseteq Q_{n+1}^{(i)}\subseteq\cdots$$

is rising chain of right ideals of R_i, by the proof of 1), there exists an integer $m_i(\geqslant 0)$ such that for any $j\geqslant m_i$

$$Q_j=Q_{j+1}.$$

Take $m=\max\{m_i\,|\,i=1,2,\cdots,n\}$, then if $j\geqslant m, Q_j^{(i)}=Q_{j+1}^{(i)}$ for any i.

Therefore $Q_j=Q_{j+1}$. So the theorem is correct.

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