Some Notes About the Cardinals of Metric Space*

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Theorem 1 Let X be a nonempty countable set, $K=\{\langle X,d\rangle: \langle X, d\rangle$ is a discrete metric space$\}$, define $\langle X,d\rangle \cong \langle X,d'\rangle$ iff$(\exists f)$(f is an equilong isomorphism from $\langle X, d\rangle$ to $\langle X,d'\rangle$), for a given $\langle X,d\rangle \in K$, define $\widehat{\langle X,d\rangle}=\{\langle X,d'\rangle \in K: \langle X,d'\rangle \cong \langle X,d\rangle\}$. Let $C=\{\widehat{\langle X,d\rangle}: \langle X,d\rangle \in K\}$, then $|C|=|K|=|\{d: d$ is a metric on X$\}|=2^{\aleph_0}$

The Theorem 2 illustrates that there exists a nonempty countable set X on which we can define $2^{\aleph_0}$ nondiscrete metric spaces such that they are not isomorphic each other.

Theorem 2 Let Q be the set of all rational numbers. Let $K_1=\{\langle Q,d\rangle: \widehat{\langle Q,d\rangle}$ is a nondiscrete metric space$\}$. Define $\widehat{\langle Q,d\rangle}$ such as Th. 1. Let $C_1=\{\widehat{\langle Q,d\rangle}: \langle Q,d\rangle \in K_1\}$, then $|C_1|=|K_1|=|\{d: d$ is a metric on Q$\}|=2^{\aleph_0}$.

The Theorem 3 shows that if X is an infinite set, then we can define $2^{|X|}$ discrete spaces on X, they are all isomorphic.

Theorem 3 Let X be an infinite set, $K_2=\{\langle X, d\rangle: \langle X, d\rangle$ is a discrete metric space$\}$. Define $\widehat{\langle X,d\rangle}$ such as Th. 1. then $\sup\{|\widehat{\langle X,d\rangle}|: \langle X,d\rangle \in K_2\}=\max\{|\widehat{\langle X,d\rangle}|: \langle X,d\rangle \in K_2\}|=|K_2|=|\{d: d$ is a metric on X$\}|=2^{|X|}$

In the proof of Th. 3, the GCH and AC were used.

References

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