A Note on Self-Mapping of the Interval*

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In this note we showed by construct a example that for $f\in C^\circ(I\,I)$, the conditions $p(f)\subset\{2^n\mid\forall n\geqslant o\}$ and $P(f)=\Omega(f)$ are not equivalent.

Let $f\in C^\circ(I\,I)$, define $\tilde{f}:[0,1/3]\to[0,1/3]$ by $\tilde{f}(x)=1/3f(3x)$, for any $x\in[0,1/3]$. We can construct a example as following:

Let $f_1\in C^\circ(I\,I)$ be identity, define $f_2\in C^\circ(I\,I)$ by

$$f_2(x)=\begin{cases}\tilde{f}_1(x)+2/3 & x\in[0,1/3]\\ -3x+2 & x\in[1/3,2/3]\\ x-2/3 & x\in[2/3,1];\end{cases}$$

then define $f_3\in C^\circ(I\,I)$ by

$$f_3(x)=\begin{cases}\tilde{f}_2(x)+2/3 & x\in[0,1/3]\\ -\frac{7}{3}x+\frac{14}{9} & x\in[1/3,2/3]\\ x-2/3 & x\in[2/3,1];\end{cases}$$

for any positive integer $n\geqslant3$ inductive define

$$f_{n+1}(x)=\begin{cases}\tilde{f}_n(x)+2/3 & x\in[0,1/3]\\ -\frac{7}{3}x+\frac{14}{9} & x\in[1/3,2/3]\\ x-2/3 & x\in[2/3,1];\end{cases}$$

Theorem $\{f_n\}_{n=1}^{\infty}$ converge uniformly to a mapping $f\in C^\circ(I\,I)$ and f satisfy $p(f)\subset\{2^n\mid\forall n\geqslant0\}$.

However, it is not difficult to show $0\notin P(f)$ but $0\in\Omega(f)$, so $P(f)\neq\Omega(f)$.

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