

The work of Lu Jia-xi*

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Lu Jia-xi was a high school teacher at the ninth middle school in Baot'ou China. The only work of his published in western journals were six brilliant papers published in the Journal of Combinatorial Theory-entitled on Lange Sets of Disjoint Triple Systems I, -VI, [LuI-VI]. This problem in one form or another dates back to the middle of the last century. It asks "when can the set of all three element subsets of set whose cardinality is congruent to 1 or 3 modulo 6 be partitioned into Steiner Triple Systems?". Lu Jia-Xi showed this could be done for all $v \not\equiv 7 \pmod{6}$ except for possibly $v = 141, 283, 521, 759, 1501, 2365$. Shortly before his tragic death he told me he had shown the existence for these cases too and would soon write up those results. [Lu7].

I will give a brief sketch of the history of this problem-the important thing to note is that the leaders in the field from Bays in 1917 through M. Hall, J. Doyen, C. C. Lindner, A. Rosa, L. Tierlink, and R. M. Wilson all tried their hand at it.

A Steiner Triple System on v points is a pair (V, B) such that B is a set of three element subset of V , and $|V| = v$ so that every pair of elements from V is contained in exactly one element of B . $D(v)$ is the largest number of disjoint Steiner Triple Systems possible on a set v . [$D^*(v)$ is largest set of isomorphic disjoint systemt on the set]. It is clear that $D(v) \leq v - 2$. We summarize the history in the following table:

| Author | Result | Year | Reference |
|---------|---|------|-----------|
| Cayley | $D^*(7) = D(7) = 2$ | 1850 | Ca |
| Kirkman | $D^*(9) = D(9) = 7$ | 1870 | Ki |
| Bays | Conjecture $D(v) \geq \frac{v-1}{2}$ | 1917 | Ba |
| Boyen | $D^*(6t+3) \geq 4t+1, 2t+1 \equiv 0(3)$ | 1972 | Do |
| | $D^*(6t+3) \geq 4t-1, 2t+1 \equiv 0(3)$ | | |
| | $D^*(6t+1) \geq t/2, t \equiv 0(2) \quad D^*(6t+1) \geq t, t \equiv 1(2) \quad D(2v+1) \geq D(v) + 2$ | | |

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|-----------|--|------|---------------------------------------|
| Tierlink | $D(3v) \geq 2v + D(v)$ $D(3^n) = 3^n - 2$ | 1973 | Ti |
| Schreiber | $D(v) = v - 2$ when $v - 2 = \pi p_i^a$ and $\text{ord}(-2) \bmod p \equiv 2(4)$ | 1973 | Sc |
| Denniston | $D(v) = v - 2$, $v = 13, 15, 19, 21, 25$ 33, 43, 49, 61, 69 | 1974 | Do |
| Rosa | For $v > 3$ $D(2v+1) = v+1 + D(v)$ Thus if $D(v) = v - 2$, $D(2v+1) = 2v - 1$ | 1974 | Ro |
| Lu | $D(v) = v - 2$, $v \neq 7$ | 1983 | Tu1, Lu2, Lu3, Lu4, Lu5, Lu6, Lu7, |

Lu Jia-Xi should have had a distinguished career as a mathematician. I am told by Zhu Lie, who is visiting at University of Waterloo that Lu solved the existence of Kirkman Systems and had an asymptotically stronger result than that of Ray-Chaudhuri and Wilson [Ra] but was not permitted to publish. He, however spent too many years as a high school teacher with virtually no time to do research and virtually no contact with the research community. When he was at last recognized and the processes of transferring him to the university were in progress he died suddenly at the age of 48. These six papers are an indication of what he could have produced if circumstances had permitted. They thus are a fitting memorial.

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