第5卷 第4期 数学研究与评论 Vol.5 No.4
1985年10月 JOURNAL OF MATHEMATICAL RESEARCH AND EXPOSITION Oct. 1985

A Topological Degree for Multivalued Generalized A-proper Mappings*

Min Le Quan(闵乐泉)

(Beijing University of Iron and Steel Technology)

The aim of this note gives a generalization of the degree theory for single valued G-A-proper mappings of [4] and a degree theory for multivalued G-A-proper mappings. Let (X, Y) be a pair of separable Banach spaces with given approximation scheme $\{X_n, Y_n, Q_n\}$ ([2] Section2). Let D be an open bounded subset of X.

Definition 1 A convex valued mapping $T: \bar{D} \to 2^Y$ is said to be Generalized A-proper (i. e. G-A-proper) with respect to $\{X_n, Y_n, Q_n\}$ if

(i) $T_n: \bar{D}_n \to 2^{Y_n}$ is an u. s. c. mapping with compact values for all $n \in N$, where $D_n = D \cap X_n$ and $T_n = Q_n T|_{X_n}$

(ii) For any sequence $\{x_{n_j} | x_{n_j} \in X_{n_j}\}$, $n_j \to \infty$, T satisfies

a) If $x_{n_j} \in \partial D_n = \partial D \cap X_{n_j}$ and $z_{n_j} \in T_{n_j}(x_{n_j})$ such that $\| z_{n_j} - Q_{n_j} y \| \to 0$ for some $y \in Y$. Then exists a $x \in \partial D$ such that $y \in Tx$;

b) If $x_{n_j} \in D_{n_j} = D \cap X_{n_j}$ and $z_{n_j} \in T_{n_j}(x_{n_j})$ such that $\| z_{n_j} - Q_{n_j} y \| \to 0$ for some $y \in Y$. Then exists a $x \in \bar{D}$ such that $y \in Tx$.

Definition 2 $T: \bar{D} \to 2^Y$ is said to be "p point" G-A-proper with respect to $\{X_n, Y_n, Q_n\}$ if T satisfies Definition 1 (i) and (ii) at piont p.

Definition 3 Let $T: \bar{D} \to 2^Y$ be G-A-proper and p be a point such that $p \notin T(\partial D)$. We define deg(T, D, p) the degree of T on D over p with respect to a given scheme, as follows. Let $\widehat{Z}$ be the set of all integers together with $\{+\infty\}$ and $\{-\infty\}$. Then $\deg(T, D, p) = \{\gamma \in \widehat{Z} |$ there exists an infinit sequence $\{n_j\}$ of positive integers with $n_j \to \infty$ such that $\deg(T_{n_j}, D_{n_j}, Q_{n_j} p) \to \gamma\}$, where $\deg(T_{n_j}, D_{n_j}, Q_{n_j} p)$ is the degree defined by Cellina and Lasota[1].

Theorem 1 Let D be an open bounded subset of X and let T: $\bar{D} \to 2^Y$ be G-A-proper with respect to a given approximation scheme, with $\|Q_n\| \leqslant M$ for all n. Let $p \notin T(\partial D)$. Then the following results hold.

*Received Mar. 15, 1983.

(i) If $\deg(T, D, p) \neq \{0\}$, then there exists an element $x \in D$ such that $p \in T(x)$.

(ii) Let $H: \bar{D} \times [0, 1] \to 2^Y$ be a mapping such that $H_t = H(\cdot, t)$ is a family of G-A-proper mappings if

(a) $T_n: \bar{D}_n \times [0,1] \to 2^{Y_n}$ is an u.s.c. mapping.

(b) If exist $\{n_j\}$, $\{x_{n_j}\}$ and $\{t_j\} \subset [0, 1]$ with $n_j \to \infty$ as $j \to \infty$, $t_j \to t_0 \in [0, 1]$, $x_{n_j} \in \partial D_{n_j}$ and $Q_{n_j} p \in Q_{n_j} H_{t_j}(x_j)$, then we get $x \in \partial D$ such that $p \in H(x, t_0)$.

(c) $p \notin H(\partial D \times [0, 1])$.

Then $\deg(H_t, D, p)$ is independent of t in $[0, 1]$.

(iii) Let $D = D_1 \cup D_2$, $D_1 \cap D_2 = \emptyset$. Put $D' = \partial D_1 \cup \partial D_2$ and suppose that $p \notin T(D')$. Then

$$\deg(T, D, p) \subseteq \deg(T, D_1, p) + \deg(T, D_2, p),$$

with equality holding if either $\deg(T, D_1, p)$ or $\deg(T, D_2, p)$ is a singleton.

(iv) If D is symmetric about the origin in X and T is a mapping, odd on ∂D, then $\deg(T, D, 0)$ is odd so that the equation $0 \in T(x)$ has a solution in D.

(v) Let T, S: $\bar{D} \to 2^Y$ be G-A-proper mappings, such that $T(x) = S(x)$ for all $x \in \partial D$. Let $p \in Y$ be a point such that $p \notin T(\partial D) = S(\partial D)$. Then $\deg(T, D, p) = \deg(S, D, p)$.

Definition 4 A mapping $T: X \to 2^{X^*}$ is said to satisfy condition(*) at point $p \in X^*$ if for any sequence $\{x_{n_j}\}$, $x_{n_j} \in \bar{D}_{n_j} \subset X_n$, and $z'_{n_j} \in T_{n_j}(x_{n_j})$ such that $\|z'_{n_j} - Q_{n_j} P\| \to 0$, $n_j \to \infty$, then $\{\|z_{n_j}\|\}$, $z'_{n_j} = Q_{n_j} z_{n_j}$ are uniformly bounded with respect to n_j.

Theorem 2 Let T: $X \to 2^{X^*}$ be mapping of type (M) ([3]) and (X, X^*) be a pair of reflexive separable Banach spaces that has given approximation scheme $\{X_n, Y_n, Q_n\}$. Let D be an open bounded subset of X and $\bar{D}$ be weakly closed. If $p \in X^* \setminus T(\partial D)$ and T satisfies the condition (*) at point p, then T is either G-A-proper mappings at point p or the equation $Tx = p$ must have a solution in D.

Theorem 3 Let a mapping T: $X \to 2^{X^*}$ be of type (M) and (X, X^*) be a pair of reflexive separable Banach spaces that has given approximation scheme $\{X_n, Y_n, Q_n\}$. If $\bar{B}(0, R) \subset X$, $0 \in X^* \setminus T(\partial B(0, R))$ and T is odd on ∂D. In addition, T satisfies condition(*) at point o. Then the equation $Tx = o$ has a solution in $B(o, R)$.

Reference

[1] Cellina. A. and Lasota, A., *Atti Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Natur.*, (8) 47 (1969), pp 434-440.

[2] Massabó, I. and Nistri, P., (Cosenza), *Bollettino U. M. I.*, (5) 13-B(1976), pp 672-685.

[3] Pascali, D. and Sburlan, S., Nonlinear mappings of Monotone type, Ed. Academiei Romania, Sijthoff & Noordhoff Intern. Publ., (1978).

[4] Min Le Quan, A Topological Degree for Generalized A-proper Mappings, *J. Math. Research and Exposition*, vol. 5, No. 2(1985), pp.59-64.