Note on the Algebraic Precision of Reducing-Dimensionality Expansion*

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As is known in [1], a general reducing-dimensionality expansion is

$$\int_{V_n} F(x)dx = \frac{1}{m_1!\cdots m_n!}\sum_{i=1}^{n}\sum_{k=1}^{m_i-1}(-1)^{\overbrace{m_1+\cdots+m_{i-1}}^{i-1}+k}\int_{\partial V_n} D^{(\overbrace{m_1,\cdots,m_{i-1}}^{i-1}k,0,\cdots 0,)}F\cdot$$
$$D^{(0,\cdots,0,m_i-k-1,m_{i+1},\cdots,m_n)}P\left(\frac{\partial x_i}{\partial\gamma}\right)ds+\rho_m(F). \tag{1}$$

In formula (1) we can choose an auxilliary polynomial $P(x)$ from the class of polyncmials $K_m=\{P_m(x)\mid P_m(x)\in\boldsymbol{P}_m$ and the coefficient of the term $x_1^{m_1}x_2^{m_2}\cdots x_n^{m_n}$ is 1, $m_1+m_2+\cdots+m_n=m\}$, in which $\boldsymbol{P}_m$ denotes the space of all polynomials of degree$\leqslant$m.

Theorem For any bounded region in $\boldsymbol{R}^n$ and any given positive integer m, there exists a class of auxilliary polynomials $P(x)\in K_m$ in (1) such that the reducing-dimensionality expansion (1) is of the highest algebraic precision 2m-1.

Proof Let $M(n,m)=\binom{n+m}{m}$. Let $\{\phi_i\}_{i=1}^{M(n,m)}$ be all monomials of degree$\leqslant m$ in $\boldsymbol{P}_m$. Without loss of generality, we can assume $\phi_{M(n,m)}=x^\alpha$, $|\alpha|=m$. $\dot{\boldsymbol{P}}_m=\text{span}\{\phi_1,\cdots,\phi_{M(n,m)-1}\}$. Let $P_m(x)=x^\alpha+\sum_{i=1}^{M(n,m)-1}a_i^{(\alpha)}\phi_i\in K_m$. By means of $P_m\perp\dot{\boldsymbol{P}}_m$ we can determine all coefficients $a_i^{(\alpha)}$, hence $P_m(x)$ is determined.

If the auxilliary polynomial $P(x)$ is $P_m(x)$ mentioned above, then we can verify that the formula (1) is of the highest algebraic precision 2m-1. Hence the proof of theorem is completed.

Reference

[1] Hsu, L. C. and Yang, J. X., Numerical Mathematics (A Journal of Chinese Universities), 4(1981), pp. 361—369.

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