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Partial Stability of Large-Scale Systems *

Ye Dawei (叶大卫)

(Institute of Mathematics, Henan Province)

Ⅰ. Introduction

It is known that in the analysis of large-scale systems there lots results on stability heve been obtained by using two different approaches concerning Liapunov functions. The first approach, whose representative works might be [1—5], uses vector Liapunov functions. The second approach, whose representaive works might be [6,7,4,8,], uses scalar functions consisting of weighted sums of scalar Liapunov functions of subsystems.

It is remarkable that almost of these stability results always involve all the state variables of composite systems. However, in many practical large-scale systems roles of the state variables are often not equal each other, and we are actually interested in knowing the behavior of only some of the state variables. On the other hand, sometimes we can obtain information about some of the var riables, but we can not for the remaining state variables. So the consideration of partial stability problems is useful both in theory and practice.

In this paper a sufficient condition for the partial stability of large-scale systems is established. The partial stability of large-scale systems is investigated in terms of their subsystems and their interconnecting structures.

In [9] the partial stability of large-scale systems is first considered. However, our criterion established here is more general than that obtatned in [9]。

Ⅱ. Main Result

Let us consider a composite, continuous dynamic system described by the vector differential equations

$$\dot{z}_i = f_i\ (z_i, t) + g_i\ (z_1, \cdots z_m, t\ ) \qquad (i = 1, \cdots,\ m\ ) \tag{1}$$

where $z_i \in \mathrm{R}^{n_i}$, R^{n_i} denotes the Euclidean n_i - space with the usual norm. f_i: $\mathrm{R}^{n_i} \times \mathrm{J} \to \mathrm{R}^{n_i}$, g_i: $\mathrm{R}^{n_1} \times \cdots \times \mathrm{R}^{n_m} \times \mathrm{J} \to \mathrm{R}^{n_i}$ and $\mathrm{J} = [\tau, +\infty)$, τ being a real number. Suppose $z = (z_1^{\mathrm{T}}, \cdots, z_m^{\mathrm{T}})^{\mathrm{T}}$.

We assume that system (1) satisfies conditions sufficient to guarentee the existence, uniqueness of solutions. Furthermore, we suppose $f_i\,(\mathrm{o}, \mathrm{t}) \equiv 0$, $g_i\,(\mathrm{o}, \mathrm{t}) \equiv$

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$0, t \in J$, $i=1, \cdots, m$, so that system (1) has the null solution.

System (1) may be viewed as a nonlinear, time-varying interconnection of m isolated subsystems

$$\dot{z}_i = f_i\ (z_i, t) \qquad (i=1, \cdots, m) \tag{2}$$

Suppose $z_i = (x_i^{\mathrm{T}}, y_i^{\mathrm{T}})^{\mathrm{T}}$ $(i=1, \cdots, m)$, where $x_i \in \mathbf{R}^{l_i}$, $y_i \in \mathbf{R}^{k_i}$, $k_i + l_i = n_i$. Without lossing generality we only consider stability properties with respect to $\xi = (x_1, x_2, \cdots, x_m)$.

First, we recall that the null solution is sald to be ξ-stable if $\forall \varepsilon > 0$, $\exists \delta = \delta(\varepsilon, t_0)$ such that $\|z_0\| < \delta \Rightarrow \|\xi(z_0, t_0, t)\| < \varepsilon$, $\forall t \geq t_0 \in J$, and asymptotically ξ-stable if it is ξ-stable and there exists a number $\sigma(t_0) > 0$, such that

$$\|z_0\| \leqslant \sigma_0 \Rightarrow \xi(z_0, t_0, t) \to 0 \quad \text{as } t \to +\infty.$$

Then, we give some preminaries.

Definition I A real-valued function $\varphi(r)$: $[0, +\infty) \to \mathbf{R}'_+$ is said to be of class K $(\varphi \in \mathrm{K})$ if it is continuous, strictly increasing and $\varphi(0)=0$.

Definition 2 Isolated subsystem (2) possesses Property A if there exist a continuously differentiable function V_i: $\mathbf{R}^{n_i} \times \mathbf{J} \to \mathbf{R}'_+$, two radially unbounded func tions $\varphi_{i_1}, \varphi_{i_2} \in \mathrm{K}$, and a function $\varphi_{i_3} \in \mathrm{K}$, such that the conditions

(i) $\varphi_{i_1}(\|x_i\|) \leq V_i(z_i, t) \leq \varphi_{i_2}(\|x_i\|)$, (ii) $V_{i(2)} \leq -\varphi_{i_3}(\|x_i\|)$

hold for $z_i = (x_i^{\mathrm{T}}, y_i^{\mathrm{T}})^{\mathrm{T}} \in \mathbf{R}^{n_i}$ and for all $t \in J$.

Now we are able to give our criterion.

Theorem. The null solution of (1) is asymptotically ξ-stable in the large if the following condition are satisfied:

(i) each isolated subsystem (2) possesses Property A;

(ii) for $\forall i \in \{1, 2, \cdots, m\}$ there exist bounded functions $\eta_{ij}(z, t)$: $\mathbf{R}^n \times \mathbf{J} \to \mathbf{R}^1$ such that

$$(\mathrm{grad}\ V_i)^{\mathrm{T}} g_i\ (z, t) \leq [\varphi_{i_3}(\|x_i\|)]^{\frac{1}{2}} \sum_{j=1}^{m} \eta_{ij}(z, t) [\varphi_{j_3}(\|x_j\|)]^{\frac{1}{2}};$$

(iii) there exists an m-vector $a = (a_1 \cdots, a_m)^{\mathrm{T}}$, $a_i > 0$, $i=1, 2, \cdots, m$, such that the matrix $\mathrm{S} = (s_{ij})$ defined by

$$s_{ij} = \begin{cases} -a_i + a_i \eta_{ii}(z, t) & if\ i=j \\ (a_i \eta_{ij}(z, t) + a_j \eta_{ji}(z, t))/2 & if\ i \neq j \end{cases}$$

is negative definite.

Proof. Let $V(z, t) = \sum_{i=1}^{m} a_i V_i(z_i, t)$. Then

$$\sum_{i=1}^{m} a_i \varphi_{i1}(\|x_i\|) \leq V(z, t) \leq \sum_{i=1}^{m} a_i \varphi_{i_2}(\|x_i\|).$$

From hypotheses it follows that $V(z,t)$ is positive definite, decrescent, and radially unbounded, and

$$\dot{V}_{(1)} = \sum_{i=1}^{m} \left\{ a_i \left[\frac{\partial V_i(z_i,t)}{\partial t} + (\text{grab } V_i)^T f_i(z_i,t) \right] + \right.$$

$$\left. + a_i \ (\text{grad } V_i)^T g_i \ (z,t) \right\} = \sum_{i=1}^{m} \left\{ a_i \dot{V}_{i(2)} + a_i \ (\text{grad } V_i)^T g_i(z,t) \right\}$$

$$\leq \sum_{i=1}^{m} a_i \left\{ -\varphi_{i_3}(\|x_i\|) + [\varphi_{i_3}(\|x_i\|)]^{\frac{1}{2}} \sum_{j=1}^{m} \eta_{ij}(z,t)[\varphi_{i_3}(\|x_j\|)]^{\frac{1}{2}} \right\}$$

Let $w^T \triangleq ([\varphi_{i_3}(\|x_1\|)]^{\frac{1}{2}} \cdots, [\varphi_{m_3}(\|x_m\|)]^{\frac{1}{2}})$, we obtain that

$$\dot{V}_{(1)} \leq w^T S w,$$

this implies S is symmetric, that

$$\dot{V}_{(1)} \leq \lambda_{\max}(S) W^T w = \lambda_{\max} \sum_{i=1}^{m} \varphi_{i3}(\|x_i\|),$$

where $\lambda_{\max}(S)$ denotes the largest eigenvalue of S and since S is negative definite, $\lambda_{\max}(S) < 0$. Therefore, there exist three functions $\varphi_1, \varphi_2, \varphi_3$ of class K such that

$$\varphi_1(\|\xi\|) \leq V(z,t) \leq \varphi_2(\|\xi\|) \text{ and } \dot{V}_{(1)} \leq -\varphi_3(\|\xi\|), \ \forall \ (z,t) \in R^n \times J.$$

Hence the null solution of (1) is asymptotically ξ-stable by virtue of extension to partial stability [10] of the Liapunov theorem on asymptotic stability.

Ⅲ. Example

As an example let us consider the system

$$\begin{cases} \dot{x}_1 = -x_1(1+y_1^2) - 2x_2 e^{\frac{1}{1+y_2^2}}, & \dot{y}_1 = y_1 + f(x_1, y_1, x_2, y_2), \\ \dot{x}_2 = -x_2 + y_2 x_2 + x_1, & \dot{y}_2 = (1+y_2^2) + y_2(x_1 - y_1)^2, \end{cases} \tag{3}$$

where $x_1, y_1, x_2, y_2 \in R^1$ and $f: R^4 \to R^1$ is a continuous and locally Lipschitzian function. System (3) may be viewed as a nonlinear interconnection of the following isolated subsystems:

$$\dot{x}_1 = -x_1(1+y_1^2), \quad \dot{y}_1 = y_1 \tag{4}$$

and

$$\dot{x}_2 = -x_2 + y_2 x_2, \quad \dot{y}_2 = (1+y_2^2)^2. \tag{5}$$

Now assume $V_1 = x_1^2$, $V_2 = x_2^2 e^{\frac{1}{1+y_2^2}}$, we have $x_1^2 \leq V_1 \leq x_1^2$, $x_2^2 \leq V_2 \leq e x_2^2$ and

$$\dot{V}_{(4)} = -2x_1^2(1+y_1^2) \leq -2x_1^2, \quad \dot{V}_{2(5)} = -2x_2^2 e^{\frac{1}{1+y_2^2}} \leq -2x_2^2.$$

Letting $V = V_1 + 3V_2$ obtains

$$\dot{V}_{(3)} = \dot{V}_{1(3)} + \dot{V}_{2(3)} = -2x_1^2 + 2x_1 x_2 e^{\frac{1}{1+y_2^2}} - 6x_2^2 = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}^T S \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

where $S = \begin{bmatrix} -2 & +e^{\frac{1}{1+y_2^2}} \\ +e^{\frac{1}{1+y_2^2}} & -6 \end{bmatrix}$, and it is negative definite. By the foregoing theorem, the null solution of (3) is asymptoticall stable with respect to (x_1, x_2).

Remark. The foregoing example can not be solved by using the criterion proposed in [9], but the example proposed in [9] can be solved by using our criterion.

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