J. Math. Res. & Exposition, 1987, No. 2

Approximation-Transforming Theory and Pansystems Approximation Theory(II)*

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Theorem 1. If B is consisting of $y=y_{\pm}(x)$, $x=a_{\pm}$, where $y'_{\pm}(x)$ are bounded, $y_{+}(x)>y_{-}(x)$, Let H^*_λ be $C(\overline{D})\cap L_2^{(1)}(D)\cap K$,

$$K:\quad \left\{\int_{y_-(x)}^{y_+(x)}\left(\frac{\partial g}{\partial y}\right)^2 dy\right\}^{1/2}\le\lambda,$$

then $\|g\|_q\le c\|g\|(\log(\lambda/\|g\|))^a$ for $g\in H^*_\lambda$, $q\ge 2$, $a=(q-2)/q$, where

$$\|g\|=\left\{\iint_D\left[A_1\left(\frac{\partial g}{\partial x}\right)^2+A_2\left(\frac{\partial g}{\partial y}\right)^2+A_3g^2\right]dxdy+\int_B\sigma(s)g^2(s)ds\right\}^{1/2},$$

$A_i>0$, $A_i\in C(\overline{D})$, $\sigma(s)>0$, $\sigma(s)\in C(B)$.

Theorem 2. Let A(D) be the family of analytic functions in/on D, $r_1<r_2<r_3$, $1\le p\le\infty$, f $f\in L^p(|z|=r_1)$, $f_\lambda\in A(|z|\le r_3)$, $\|f_\lambda\|\le\lambda$, $\|*\|$ be the norm in $L^p(|z|=r_3)$. If $\|f-f_\lambda\|^*\le\varepsilon(t)\downarrow 0$, $\int t^{a-1}[\varepsilon(t/u)]^b\,dt<\infty$, $u>1$, $a=(\log r_2-\log r_1)/(\log r_3-\log r_1)$, $b=(\log r_3-\log r_1)/(\log r_3-\log r_1)$, $\|*\|^*$ is the norm in $L^p(|z|=r_1)$, then f is equal to a $g\in A(r_1<|z|<r_2)$ almost everywhere on $|z|=r_1$, and the boundary values $g\in L^p(|z|=r_2)$,

$$\|g-f_\lambda\|^{**}=O\left\{\int_\lambda^\infty t^{a-1}\left(\varepsilon\left(\frac{t}{u}\right)\right)^b dt\right\}.$$

where $\|*\|^{**}$ is the norm in $L^p(|z|=r_1)$.

Theorem 3. Let $f\in L^p(\mathrm{Re}\,z=r_1)$, $f_\lambda\in A(r_1\le\mathrm{Re}\,z\le r_3)$, $\|f_\lambda\|\le\lambda$, $\|*\|$ be the norm in $L^p(\mathrm{Re}\,z=r_3)$. If $\|f-f_\lambda\|^*\le\varepsilon(\lambda)\downarrow 0$, $\int^\infty t^{a-1}[\varepsilon(\frac{t}{u})]^b dt<\infty$, $u>1$, $a=(r_2-r_1)/(r_3-r_1)$, $b=(r_3-r_2)/(r_3-r_1)$, $\|*\|^*$ being the norm in $L^p(\mathrm{Re}\,z=r_1)$, then there exists $g\in A(r_1<\mathrm{Re}\,z<r_2)$, whose boundary values g are equal to f almost everywhere on $\mathrm{Re}\,z=r_1$, and $g\in L^p(\mathrm{Re}\,z=r_2)$,

$$\|g-f_\lambda\|^{**}=O\left\{\int_\lambda^\infty t^{a-1}\left[\varepsilon\left(\frac{t}{u}\right)\right]^b dt\right\}.$$

where $\|*\|^{**}$ is the norm in $L^p(\mathrm{Re}\,z=r_2)$.

References

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* *Received Jan. 15, 1987.*