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Two Wider Classes of Combinatorial Identities*

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Here introduced are two general classes of combinatorial identities that may be defined via inverse relations due to Gould and Hsu (cf. Duke Math J. 40 (1973), 885–891).

Classes (Σ) **and** (Σ^*). Any identity is said to belong to the class (Σ) if it can be embedded in either of the forms (reciprocal relations)

$$f_n = \sum_{k=0}^{n} (-1)^k \binom{n}{k} \varphi(k, n) \cdot g_k \tag{1}$$

$$g_n = \sum_{k=0}^{n} (-1)^k \binom{n}{k} (a_{k+1} + k b_{k+1}) \varphi(n, k+1)^{-1} \cdot f_k \tag{2}$$

where $\{a_k\}$ and $\{b_k\}$ are any two sequences of real or complex numbers such that $\varphi(x, n) = \prod_{i=1}^{n} (a_i + b_i x) \neq 0$ for integers $n, x \geq 0$ with $\varphi(x, 0) \equiv 1$.

Similarly, an identity is said to belong to (Σ^*) if it can be embedded in either of the rotated forms of (1) and (2). (See loc. cit.)

Remark and Examples. It is not difficult to find that nearly 30% of the total 500 identities displayed in H. W. Gould's formulary „Combinatorial Identities" (Morgantown, U.S.A. 1972) can be embedded in (1) or (2) or one of their rotated forms, so that they belong to the union of set (Σ) $\cup$ (Σ^*). In particular, one may verify that the well-known/classic identities due to Abel, Hagen-Rothe, Jensen, Saalschutz, Krall, Knuth, Carlitz, and Egorychev-Liskovets (cf. G. P. Egorychev's "Integral Representation and the Computation of Combinatorial Sums", Amer. Math. Soc. Translation, Vol. 59, 1984, U.S.A.), respectively, are all members of the class (Σ). Accordingly, these identities can be either proved anew or used to yield new identities by means of the inverse relations (1) $\Leftrightarrow$ (2). Also, it may be observed that not only the well known pair of Moriarty identities and the corresponding inverses (viz. Marcia Ascher's identity and its companion piece) but also the classical identity of Van Ebbenhorst Tengbergen and its inverse are members of the class (Σ^*). (Details will appear elsewhere.)

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