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Approximation-Transforming Theory and Pansyst Approximation Theory(III)*

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Domain $D \in S_m(r,\theta)$ means $D \subset R^m$, and contains certain cone with radius r and m solid angle θ issued from every point of D.

Theorem 1 If $D \in S_m(r,\theta)$, $f \in C(D)$, $\omega(\delta, f, W_\infty^{(0)}(D)) \leqslant \lambda\delta^a$, $(\lambda^{-1}\|f\|_p)^a \leqslant r$, then for $1 \leqslant p \leqslant q \leqslant \infty$, we have $\|f\|_q \leqslant c\lambda^b\|f\|_p^d$, where $a = p/(pa+m)$, $b = me/(ap+m)$, $d = pa + p/q$, $c = (\max[(\frac{m}{\theta})^{1/p}, \frac{m}{m+a}])^e$, $e = 1-p/q$.

Theorem 2 If $D \in S_m(r,\theta)$ for certain r,θ, $g \in W_s^{(l)}(D) \cap L^p(D)$, $\|g\|_{(l)s} \leqslant \lambda$, $p \geqslant 1$, $s > 1$, then $g \in C(D)$ almost everywhere and $\|g\|_q \leqslant c\lambda^a\|g\|_p^b$, where $q \geqslant p$, $a = m(q-p)/(pqd)$, $b = p/q + (l-\frac{m}{s})(q-p)/(qd)$, $d = l - \frac{m}{s} + \frac{m}{p}$, and c is independent of λ and g.

Theorem 3 If $D \in S_m(r,\theta)$ for certain r and θ, $0 \leqslant k \leqslant l-1$, $\|g\|_{(l)p} \leqslant \lambda$, then $g \in C^{[k]}(D)$ ($g \in C^{[k]}(\overline{D})$ provided the boundary of D satisfying local Lipshitz condition) and $\|D^{(k)}g\|_\infty \leqslant c\lambda^a\|g\|_p$, where $D^{(k)}$ is any given partial derivative of k-order, and $a = (k+\frac{m}{p})/l$, $b = (l-k-\frac{m}{p})/l$, c is independent of λ and g.

References

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