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Left H-rings Whose Radicals Are Not Periodic*

Wang Chengde (王成德)

(Beijing Institute of Technology)

An associative ring (algebra) in which every subring (subalgebra) is a left ideal is called a left H-ring (left H-algebra). In [1], left H-algebras are charcterized and the problem to characterize left H-rings is proposed. In this paper, we characterize left H-rings whose radicals are not periodic.

Let R and R' be two rings. If R is isomorphic to R' we write $R=R'$. The direct sum of R and R' is denoted by $R\oplus R'$. The direct sum of the The additive groups of R and R' is denoted by $R+R'$. If the additive group of R is periodic we call R a periodic ring. For $x\in R$, the additive subgroup of R generated by x is denoted by $\{x\}$ and its order is denoted by $o(x)$.

The result of this paper is the following:

Theorem. Suppose R is a ring and its nil radical is not a nonzero pe periodic ring. Then R is a left H-ring if and only if one of the following conditions holds.

(1) $R=\sum_{p_i\in I}\oplus F_{p_i}$ where F_{p_i} is the prime field of order p_i and I is a set of some primes.

(2) $R=\{r_0\}$ where $o(r_0)=\infty$, $r_0^2=nr_0$, n is a nonzero integer.

(3) $R=R_1\oplus\sum_{p_i\in I}\oplus F_{p_i}$ where $R_1=\{r_0\}$, $o(r_0)=\infty$, $r_0^2=nr_0$, $n\neq 0$ is an integer. F_{p_i} is the prime field of order p_i and I is a set of some primes which divides n.

(4) $R=\sum_{q_j\in J}R_{q_j}$ where each R_{q_j} is an ideal of R and $R_{q_i}R_{q_j}=0$ for $q_i\neq q_j$ and J is a set of some primes. Let N_{q_j} be the annihilator of R_{q_j}, then each R_{q_j} satisfies one of the following conditions.

(a) $R_{q_j}=N_{q_j}$;

(b) $R_{q_j}=\{a\}+N_{q_j}$ where q_ja, $a^2\in N_{q_j}$ and $o(a^2)=q_j$;

(c) $R_{q_j}=\{a,\ b\}+N_{q_j}$ where q_ja, q_jb, $a^2\in N_{q_j}$, $o(a^2)=q_j$ and there are i

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only small c_1, c_2 and big b_1, b_2, as shown in (5), but also small a_1, a_2, D_1, , D_2, β_1, β_2^* and big β_1^*, β_2 can ensure the global stability.

References

[1] J. M. Mahaffy and C. V. Pao, Models of genetic control by repression with time delays and spatial effects, J. Math. Biology, 20(1984), 39—57.

[2] C. V. Pao and J. M. Mahaffy, Qualitative analysis of a coupled reaction-diffusion model in biology with time delays, J. Math. Anal. Appl., 109 (1985), 355—371.

(from 106)

integers n, m, m' such that $b^2 = na^2$, $ab = ma^2$, $ba = m'a^2$ and for which the congruence equation $X^2 + (m + m')X + n \equiv 0 \pmod{q_j}$ has no integer solution X.

(5) $\mathbf{R} = \mathbf{P} \oplus \mathbf{K}$ where $\mathbf{P} = \sum_{p_i \in \mathbf{I}} \oplus \mathbf{F}_{p_i}$ satisfies (1) and $\mathbf{K} = \sum_{q_i \in \mathbf{J}} \mathbf{R}_{q_j}$ satisfies (4).

(6) $\mathbf{R} = \mathbf{K} + \mathbf{R}_1$ where $\mathbf{K} = \sum_{q_i \in \mathbf{J}} \mathbf{R}_{q_j}$ satisfies (4) and $\mathbf{R}_1 = \{r_0\}$, $o(r_0) = \infty$, $r_0^2 = nr_0$ for $n \neq 0$. For each element x of $\mathbf{K}$, $xr_0 = 0$, $r_0 x = nx$.

(7) $\mathbf{R} = \mathbf{K} + \mathbf{A}$, where $\mathbf{K} = \sum_{q_i \in \mathbf{J}} \mathbf{R}_{q_j}$ satisfies (4) and $\mathbf{A} = \mathbf{R}_1 \oplus \sum_{p_i \in \mathbf{I}} \oplus \mathbf{F}_{p_i}$ satisfies (3), $\mathbf{R}_1 = \{r_0\}$, $o(r_0) = \infty$, $r_0^2 = nr_0$. For each $x \in \mathbf{K}$, $r_0 x = nx$, and for each $x \in \mathbf{K}$, $a \in \mathbf{A}$, $b \in \sum_{p_i \in \mathbf{I}} \oplus \mathbf{F}_{p_i}$, $xa = 0$, $bx = 0$.

Reference

[1] Liu Shaoxue, J.Beijing Normal Univ. (natural sci.), 1979, no. 3, 1—6.