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The Star-Kernel for a Quasidifferentiable Function in One-Dimensional Space

Gao Yan (高 岩)
(Dalian Institute of Technology)

Let f be a function from $\mathbf{R}^1$ into $\mathbf{R}^1$. Suppose f is quasidifferentiable at $x\in\mathbf{R}^1$. The following results have been obtained.

Lemma 1 We denote by $A:=[a_1, a_2]$ and $B:=[b_1, b_2]$ two intervals. Then $[A, B]\in\mathscr{D}f(x)$ if and only if

$$\begin{cases} a_2+b_1=f'(x;+1), \\ a_1+b_2=-f'(x;-1) \end{cases}$$

hold, where $\mathscr{D}f(x)$ denotes the quasidifferential set of f at $x\in\mathbf{R}^1$.

Theorem 2 There exists a quasidifferential $[\underline{\partial}_0 f(x), \bar{\partial}^0 f(x)]\in\mathscr{D}f(x)$ such that

$$\underline{\partial}_0 f(x)+\bar{\partial}^0 f(x)=\bigcap_{[\underline{\partial} f(x), \bar{\partial} f(x)]\in\mathscr{D}f(x)}[\underline{\partial} f(x)+\bar{\partial} f(x)]$$

$$=[\min\{f'(x;+1), -f'(x;-1)\}, \max\{f'(x;+1), -f'(x;-1)\}]$$

and

$$\bar{\partial}^0 f(x)-\bar{\partial}^0 f(x)=\bigcap_{[\underline{\partial} f(x), \bar{\partial} f(x)]\in\mathscr{D}f(x)}[\bar{\partial} f(x)-\bar{\partial} f(x)]$$

$$=[-\max\{0, -f'(x;+1)-f'(x;-1)\}, \max\{0, -f'(x;+1)-f'(x;-1)\}].$$

According to the theorem given above and [3, Th. 6], the star-kernel $\partial_{\circledast}f(x)$ and $\partial^{\circledast}f(x)$ can be expressed as $\partial_{\circledast}f(x)=\underline{\partial}_0 f(x)+\bar{\partial}^0 f(x)$ and $\partial^{\circledast}f(x)=\bar{\partial}^0 f(x)-\bar{\partial}^0 f(x)$.

Corollary 3 The directional differentiability of f is equivalent to its quasidifferentiability.

Theorem 4 The set-valued map $\partial_{\circledast}f(x)$ is upper semicontinuous at $x\in\mathbf{R}^1$ if and only if the function $a_1(x)=\min\{f'(x;+1), -f'(x;-1)\}$ is lower semicontinuous at $x\in\mathbf{R}^1$, and the function $a_2(x)=\max\{f'(x;+1), -f'(x;-1)\}$ is upper semicontinuous at $x\in\mathbf{R}^1$.

Example Let $g(x)$ be $g(x)=\begin{cases} x^{3/2}\sin 1/x & \text{if } x>0, \\ x & \text{if } x\leqslant 0. \end{cases}$ We have $\partial_{\circledast}g(0)=[0, 1]$, $\partial^{\circledast}g(0)=[-1, 1]$, and note that it is not locally Lipschitzian at $x=0$.

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ly with the clarifying operator. That just means that the precise classical mathematics is included in a higher form in the medium mathematical system MM.

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