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Dominance Theory and Plane Partitions: I. Introduction*

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Let $\bar{\lambda}\in N_0^k$ satisfying $\lambda_1\geq\lambda_2\geq\cdots\geq\lambda_k$ for natural number set N_0. An order ideal λ determined by $\bar{\lambda}$ is defined to be $\{(i,j): 1\leq j\leq\lambda_i, 1\leq i\leq k\}\subset N^2$. A plane partition (reverse) π of shape λ is an order-reversing (preserving) map from λ to N_0. If P is any set of plane partitions, the generating function (abbrev. as GF) of P is the polynomial or fromal power series $\sum_{\pi\in P}q^{|\pi|}$, where $|\pi|$ denote the weight of π. The enumeration theory of plane partitions is an active field of combinatorics. There are several approaches which have been successfully used to treat that problem such as lattice permutation method (MacMahon), basic hypergeometric function (Andrews), algorithm correspondence (Knuth) and Weyl's character formula of Lie theory (Macdonald). In this series of papers we shall use the domination method to establish the determinant expressions for the GF of plane partitions with restrictions. By means of determinant computations, a number of simplified results will be obtained which include the classicall main theorems in this field as their special cases. Contrary to the above mentioned methods, this approach may be more accessible to the readers.

For simplicity, we make the convention as follows: $\langle x\rangle_k=\prod_{i=1}^{k}(1-q^{x+i-1})$, rising q-factorial x of order k for $k\in N_0$ and $\langle x\rangle_1=\langle x\rangle$ shortly; $\begin{bmatrix} x \\ k \end{bmatrix}$, Gaussian binomial coefficient; $\langle\bar{x},\bar{y}\rangle$, scalar product for $\bar{x},\bar{y}\in N_0^k$; $\Delta(x_1,x_2,\cdots x_k)$, Vandermonde determinant of variables x_i. Based on this preparation, we can state our preliminary theorems on the evaluation of determinants which will be used as the main tools of simplification for the GF of plane partitions in the forthcoming papers of this series.

Theorem I.I

i. $\det\limits_{k\times k}|\langle r_i+j+1\rangle_{k-j}|=q^{2\binom{k+1}{3}}\Delta(q^{r_1},q^{r_2},\cdots,q^{r_k})$.

ii. $\det\limits_{k\times k}|\langle x+1\rangle_{r_i+j}^{-1}|=q^{\binom{k}{2}x+2\binom{k+1}{3}}\Delta(q^{r_1},q^{r_2},\cdots,q^{r_k})/\prod_{i=1}^{k}\langle x+1\rangle_{r_i+k}$

(to 214)

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