J. Math. Res. & Exposition, 1988, No. 2

Some Notes on a Minimization Problem*

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Abstract

Let $X \subset [a, b]$ be a compact set containing at least $n+1$ points and K an n-dimensional Haar subspace in $c[a, b]$. Let $F(x, y)$ be a nonnegative function, defined on $X \times (-\infty, \infty)$, satisfying $\|F(\cdot, p)\| < \infty$ with the L_∞ norm for some $p \in K$, where $F(x, p) \equiv F(x, p(x))$.

The minimization problem discussed in this paper is to find an element $p \in K$ such that $\|F(\cdot, p)\| = \inf_{q \in K} \|F(\cdot, q)\|$, such an element p (if any) is said to be a minimum to F in K.

The author in [1, 2] studied this problem and has given the main theorems in the Chebyshev theory under the following assumptions:

(A) $\lim_{|y| \to \infty} F(x, y) = \infty, \ \forall x \in X$; (B) $\lim_{v \to y} F(x, v) = F(x, y), \ \forall x \in X, \ \forall y$;

(C) $\varliminf_{\substack{u \to x \\ v \to y}} F(u, v) = F(x, y), \ \forall x \in X, \ \forall y$; (D) For each $x \in X$ there exist two real numbers $f^-(x)$ and $f^+(x)$, $f^-(x) \le f^+(x)$, such that $F(x, y)$ is strictly decreasing with respect to y on $(-\infty, f^-(x)]$ and strictly increasing on $[f^+(x), \infty)$, and $F(x, y) = F^*(x) := \inf_v F(x, v)$ on $[f^-(x), f^+(x)]$.

Denote $f_1(x) = \inf\{y: F(x, y) \le \|F^*\|\}$, $f_2(x) = \sup\{y: F(x,) \le \|F^*\|\}$, $\overline{f}_1(x) = \varlimsup_{u \to x} f_1(u)$, $\overline{f}_2(x) = \lim_{u \to x} f_2(u)$, $G = \{q \in K: f_1 \le q \le f_2\}$.
For $p \in K$ set $X_p = \{x \in X: F(x, p) = \|F(\cdot, p)\|\}$, $\overline{X}_+ = \{x \in X_p: p(x) \le f^-(x)\}$, $\overline{X}_- = \{x \in X_p: p(x) \ge f^+(x)\}$, $X_0 = \{x \in X_p: f^-(x) \le p(x) \le f^+(x)\}$,

$$\sigma(x) = \begin{cases} 1, & x \in \overline{X}_+ \\ -1, & \in \overline{X}_-. \end{cases}$$

A system of $n+1$ ordered points $x_1 < x_2 < \cdots < x_{n+1}$ in $\overline{X}_+ \cup \overline{X}_-$ is said to be a generalized alternation system, if it satisfies $\sigma(x_{i+1}) = -\sigma(x_i)$, $i = 1, \cdots, n$.

Theorem 1 Let $p \in K$. Then the following statements are equivalent: (a) $X_0 \ne \phi$; (b) $p \in G$; (c) $\|F(\cdot, p)\| = \|F^*\|$. Moreover, each of them implies that p is a minimum to F.

Theorem 2 Let $p \in K$. Then the following statements are equivalent:

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*Received July 10, 1985.