J. Math. Res. & Exposition, No.3, 1988

A Claas of Generalized Stirling Transforms*

L. C. Hsu (徐利治)

(Dalian University of Technology)

Here announced is an extension of the basic result presented in my fomer note (cf. The Fibonacci Quarterly, 4 (1987), 346—351).

Denote by Γ the ring of formal power series. Two elements φ and ψ are called reciprocal elements if $\varphi(\psi(t)) = \psi(\varphi(t)) = t$ with $\varphi(0) = \psi(0) = 0$.

Definition 1 A sequence of polynomials $\{p_n(t)\}$ is said to be normal if $\deg p_n(t) = n$, $p_0(t) = 1$ and $p_n(0) = 0$, $(n \geq 1)$.

Definition 2 A linear operator P is called a fundamental operator associated with the given normal sequence $\{p_n(t)\}$ if $\mathrm{P}p_0(t) = 0$, $\mathrm{P}p_n(t) = np_{n-1}(t)$, $(n \geq 1)$ and if for any given formal series $\Sigma_0^\infty c_k p_k(t)$ we have $\mathrm{P}\Sigma_0^\infty c_k p_k(t) = \Sigma_1^\infty k c_k p_{k-1}(t)$.

Inversion Theorem Let $\{p_n(t)\}$ and $\{q_n(t)\}$ be two normal sequences of polynomials associating with fundamental operators P and Q respectively. Then we have the following pair of generalized Stirling reciprocal transforms

$$y_n = \sum_{k \geq 0} \frac{1}{k!} [\mathrm{Q}^k p_n(\varphi(t))]_{t=0} x_k \qquad (1)$$

$$x_n = \sum_{k \geq 0} \frac{1}{k!} [\mathrm{P}^k q_n(\psi(t))]_{t=0} y_k \qquad (2)$$

if and only if φ and ψ are reciprocal elements of Γ, where either $\{x_n\}$ or $\{y_n\}$ is an arbitrary finite sequence of variables.

There is also a rotated form for the reciprocal relations (1) and (2), namely the letters k and n appearing in the summation kernels may be interchanged. The theorem implies a variety of interesting special cases. The simple case given by $\varphi(t) = \psi(t) = t$, $p_n(t) = t^n$, $q_n(t) = [t]_n$, $\mathrm{P} = \mathrm{D}$, $\mathrm{Q} = \triangle$ (differencing) just gives the classical Stirling transforms.

*Received May 30, 1988.