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P_1-Compact Mappings and Leary-Schauder Boundary Condition

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Let X be a Banach space and let $\Gamma_a = \{X_n, P_n\}$ be a projectionally com plete scheme. Let D be a bounded open subset of X and $T: \overline{D} \to X$ an P_1 compact mapping under the condition weaker than Leary Schauder boundary condition, we show several fixed point theorems of T and consider the rela tion between the fixed points and eigenvectors of T.

For $H \subset \overline{D}$, we write $E_H = \{\lambda > 1 : Tx = \lambda x$ for some $x \in H\}$.

Theorem 1 Let $T: \overline{D} \to X$ be a bounded continuous P_1 compact mapping and $0 \in D$ such that

(i) $\lambda \quad E_{OD} \setminus E_{\overline{D}} \cap [1, \lambda) \neq \emptyset$,

(ii) $\lambda I\text{-}T$ is locally one to one for $\lambda \geq 1$.

Then T has a fixed point in $\overline{D}$.

Theorem 2 Let X be an Π_1 Space and let $T: \overline{D} \to X$ be an 1 ball contrac tion with $0 \quad D$ such that the conditions (i) and (ii) of Theorem 1 hold. If $(I-T)(\overline{D})$ is closed in X, the T has a fixed point in $\overline{D}$.

Corollary 1 Let X be an Π_1 Space and D a bounded open subset of X with $0 \quad D$. A: $X \to X$ is bounded continuous accretive map, $F: \overline{D} \to X$ is ball-c condensing and $C: \overline{D} \to X$ is compact. If $T = F + C - A$ satisfies the conditions (i) and (ii) of Theorem 1, the T has fixed point in $\overline{D}$.

In the following, we assume that $T: X \to X$ is a bounded continuous P_1-compact map such that $\lambda I\text{-}T$ is locally one-to-one for $\lambda \geq 1$. Write $E^* = \{\lambda \geq 1:$ $Tx = \lambda x$ for some $x \quad X\}$ and $E = \{x_\lambda: \lambda \quad E^*, Tx_\lambda = \lambda x_\lambda\}$. Under the above assum ptions, we show that E^* is open in $[1, \infty)$; the map $\psi: E^* \to E$ defined by $\psi(\lambda) = x_\lambda$ is continuous and the following theorems.

Theorem 3 Under the above assumptions, T has a fixed point in X if and only if $\liminf_{\lambda \to \lambda_0} \|x_\lambda\| < \infty$ where $\lambda_0 = \inf E^*$.

Theorem 4 Under the above assumptions, if $Tx \neq x$ for all $x \quad X$, then ea each eigenvector $x \quad E$ lies in an unbounded component of E. (to 125)

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