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Dominance Theory and Plane Partitions

Ⅲ. Enumeration of Column-Strict Reverse Plane partitions

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Let $\mathrm{RF}_\lambda(\bar{n}; q)$ denote the GF for column-strict reverse plane partitions of shape λ with part restriction $\bar{n}$ (i.e., the parts in ith row do not exceed n_i). By introducing the correspondence of reverse plane partitions without and with column-strict restriction, we have the following determinant expression.

Theorem 3.1

$$\mathrm{RF}_\lambda(\bar{n}; q) = q^{n(\lambda)} \det_{k\times k} \left| \begin{bmatrix} n_i - i + \lambda_i + 1 \\ j - i + \lambda_i \end{bmatrix} q^{\binom{j-i}{2} + (j-i)\lambda_i} \right|$$

The simplified forms are as follows:

Theorem 3.2

$$\mathrm{RF}_\lambda(n\bar{I} + \bar{J} - \bar{\lambda}; q) = q^{n(\lambda)} \prod_{(i,j)\in\lambda} \frac{\langle n - c_{ij} + 1\rangle}{\langle h_{ij}\rangle}$$

Theorem 3.3 (Stanley, 1971).

$$\mathrm{RF}_\lambda(n; q) = q^{n(\lambda)} \prod_{(i,j)\in\lambda} \frac{\langle n + c_{ij} + 1\rangle}{\langle h_{ij}\rangle}$$

Corollary 3.4

$$\mathrm{RF}_{r*c}(n; q) = q^{\binom{r}{2}c} \prod_{i=1}^{r} \begin{bmatrix} n + c - i + 1 \\ c \end{bmatrix} \Big/ \begin{bmatrix} c + i - 1 \\ c \end{bmatrix}$$

And dually.

$$\mathrm{RF}_{r*c}(n; q) = q^{\binom{r}{2}c} \prod_{j=1}^{c} \begin{bmatrix} n + r - j + 1 \\ r \end{bmatrix} \Big/ \begin{bmatrix} r + j - 1 \\ r \end{bmatrix}$$

Corollary 3.5

$$\mathrm{RF}_\lambda(\infty; q) = q^{n(\lambda)} \prod_{(i,j)\in\lambda} \langle h_{ij}\rangle^{-1}.$$

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