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A Newton Method for Minimizing One-Order Lipschitz Functions

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In this paper, we consider the following unconstrained optimization problem

$$\text{(P)} \qquad \min\{f(x)\,|\,x \in \mathbf{R}^n\},$$

where $f(x)$ is a one order Lipschitz function on $\mathbf{R}^n$, i.e., $g(x)$—the gradient of $f(x)$—is Lipschitzian. We will represent a kind of Newton method for solving the problem (P).

Denoting the generalized Hessian matrix of $f(x)$ at x by $\partial^2 f(x)$, we define a ser valued mapping $N^+ : \mathbf{R}^n \to P(\mathbf{R}^n)$ by

$$N^+(x) = \{y = aH^{-1}g(x)\,|\,\forall H \in \partial^2 f(x),\ H^{-1} \text{ exists, } a = a(x, H) \text{ is determined by some methods}\}.$$

Starting from any point $x_1 \in \mathbf{R}^n$, the sequence $\{x_i\}$ generated by Newton method for solving (P) will be defined by

$$x_{i+1} \in N^+(x_i),\quad i = 1, 2, \cdots.$$

Theorem 1 Suppose that there exists a $x_0 \in \mathbf{R}^n$ such that the level set $L(x_0) = \{x\,|\,f(x) \leq f(x_0)\}$ is a bounded convex set and $f(x)$ is uniformly convex on $L(x_0)$. If $a = a(x, H)$ is the optimal step, i.e., $f(x - aH^{-1}g(x)) = \min\{f(x - aH^{-1}g(x))\,|\,a > 0\}$, then

(a) $N^+(x)$ is well defined at each $x \in L(x_0)$ and mapping N^+ is closed at each $x \in L(x_0)$.

(b) for any $x_1 \in L(x_0)$, the sequence $\{x_i\}$, generated by the above Newton method, terminates at the unique optimization solution x of (P) or converges to x^*.

Theorem 2 Suppose that x^* is a local optimization solution of (P) and $f(x)$ is twice differentiable at point x^* and uniformly convex near x^*. Then there exists a $\sigma > 0$ such that if $x_1 \in N(x^*, \sigma)$, the sequence $\{x_i\}$, which is generated by the above Newton method with stepsize $a_i = a_i(x_i, H_i) = 1$, linearly converges to x^*.

Furthermore, if the generalized Hessian matrix $\partial^2 f(x)$ satisfy the following (to 401)

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belongs to M_{n+p}.

Proof From the definition of $F(z)$ we have

$$D^{n+p-1}f(z)=D^{n+p}F(z)$$

and

$$(n+p)D^{n+p}f(z)=(n+p+1)D^{n+p+1}F(z)-D^{n+p}F(z).$$

From these relations and the fact that $f(z)\in M_{n+p-1}$, we get

$$\mathrm{Re}\left(\frac{(n+p+1)D^{n+p+1}F(z)-D^{n+p}F(z)}{(n+p)D^{n+p}F(z)}-2\right)$$

$$=\mathrm{Re}\left(\frac{D^{n+p}f(z)}{D^{n+p-1}f(z)}-2\right)<-\frac{n+p-1}{n+p}$$

from which it follows that

$$\mathrm{Re}\left(\frac{D^{n+p+1}F(z)}{D^{n+p}F(z)}-2\right)<-\frac{n+p}{n+p+1}$$

Thus $F(z)\in M_{n+p}$.

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(from 402)

condition in a neighborhood of x^*,

$$\|H-H^*\|\leq K\|x-x^*\|,\ \forall H\in\partial^2 f(x),\ H^*=\partial^2 f(x^*),$$

then the algorithm given above possesses the convergency of order 2.

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