Journal of Mathematical Research and Exposition
Vol.9, No.4, Nov., 1989

On the Convergence of BFGS Algorithm*

Guo Jian

(Dept. of Appl. Math., Dalian Univ. Tech.)

In this paper, we consider unconstrained optimization problem

(P) $$\min\{f(x)\,|\,x\in R^n\},$$

where $f(x)$ is continuously differentiable and the gradient $g(x)$ of $f(x)$ is Lipschitzian on R^n. Suppose sequence $\{x_i\}$ is generated by BFGS algoritm when it is used to the problem (P) with exact or inexact line searches, i.e., the stepsize a_i satisfy either of the following conditions

(a) $$g(x_i+a_is_i)^{\mathrm T}s_i=0, \qquad \text{(a)}$$

(b) $$\begin{cases} f(x_i+a_is_i)\leqslant f(x_i)+\rho g(x_i)^{\mathrm T}s_i \\ g(x_i+a_is_i)^{\mathrm T}s_i\geqslant \sigma g(x_i)^{\mathrm T}s_i \end{cases}$$

where $\rho\in(0,\frac{1}{2})$ and $\sigma\in(\rho,1)$ are given and s_i is a search direction, and with starting point x_1.

Under some conditions (without the second continuous differentiability of $f(x)$) which are milder than those in [3], convergence and superlinear convergence of the above algorithm are proven here.

Theorem In addition to the conditions mentioned above, suppose that there exists an $x_0\in R^n$ such that the level set $L(x_0)=\{x\,|\,f(x)\leqslant f(x_0)\}$ is a bounded convex set and $f(x)$ is convex on this set. Let x_1 be any point in $L(x_0)$. Then the BFGS algorithm terminates at an optimization solution or any cluster of $\{x_i\}$ is a optimization solution to (P).

Furthermore, if $f(x)$ is uniformly convex on $L(x_0)$ and $f(x)$ is twice differentiable at point x^* and the following condition is satisfied near x^*

$$\|H-\partial^2 f(x^*)\|\leqslant K\|x-x^*\|,\quad \forall H\in\partial^2 f(x),$$

where $\partial^2 f(x)$ is the generalized Hessian matrix of f at x and K is a positive number, then $\{x_i\}$ superlinearly converges to the unique optimization solution x^* to (P).

References

[1] J.Guo, On the Convergence of Variable Metric Methods, J.Dalian Inst.of Tech., 1988 (in press).
[2] J.Guo, A Newton Method for minimizing One Order Lipschitz Function.
[3] Powell M.J.D. Some Global Convergence Properties of variable Metric Algorithm for Minimization without exact line searches, SIAM AMS Proceedings, Vol.9, 1976.

* Received Dec.3, 1987