Journal of Mathematical Research and Exposition
Vol.10 No.1 Feb. 1990

A Condition for a Subdirectly Irreducible Ring to be a Division Ring

Wang Yu
(Siping Normal College)

Let R be an associative ring, H is the intersection of all non-zero ideals of R. If $H \neq (0)$, R is said to be subdirectly irreducible.

In this paper, we give a condition for a subdirectly irreducible ring to be a division ring. As the conclusion, we have the following theorem.

Theorem let R be a subdirectly irreducible ring with a minimal one-sided ideal, and if H has no non-zero nilpotent elements, then R is a division ring.

The following three lemmas (see [1]) is needed for the proof of the theorem:

Lemma 1 If R has a non-zero nilpotent one-sided ideal, then R has a no non-zero nilpotent ideal.

Lemma 2 If I is any minimal one-sided ideal of R, then $I^2 = (0)$, or I has an idempotent.

Lemma 3 If R has no non-zero nilpotent one-sided ideals, and if e is an idempotent element of R, that is $e = e^2 \neq 0$, then the following condition are equivalent.

(1) eR is a minimal right ideal of R.

(2) Re is a minimal left ideal of R.

(3) eRe is a division ring.

Proof of theorem: If R has a non-zero nilpotent one-sided ideal, by lemma 1, then R has a non-zero nilpotent ideal I. That is $I^n = (0)$, where $n > 1$ a fixed integer, as $H \subseteq I$, hence $H^n = (0)$, a contradiction, therefore R has no non-zero nilpotent one-sided ideals.

Supoose $L \neq (0)$ is a minimal one-sided ideal of R, by lemma 2, we have $e = e^2 \neq 0$, $e \in L$. By lemma 3, then eR is a minimal right ideal of R, Re is a minimal left ideal of R, and eRe is a division ring. Let D be the left annihillator of H in R, if eH = (0), then $e \in$ D, thus D is a non-zero ideal of R, hence $H \subseteq D$. Thus $H^2 \subseteq DH = (0)$, a contradiction, therefore $0 \neq eH \subseteq eR$. Since eR is a minimal right ideal of R, then eH = eR, hence $e \in eR = eH \subseteq H$.

* Received Jan.15, 1988.

If $xe=0$ where $x\in R$, then $(eRx)^2=(0)$. Since $eRx\subseteq H$, H has no non-zero nilpotent elements, thus $eRx=(0)$. Hence $e(x)=(0)$, where (x) is the ideal generated by x in R. If $x\neq 0$, we have $H\subseteq(x)$, thus $eH=(0)$, a contradiction, hence $x=0$, thus e is not a right zero-divisor of R. For $r\in R$ then $(re-r)e=0$, thus $re-r=0$, hence e is the right identity element of R.

As the same way, e is also the left identity element of R. Thus e is the identity element of R, hence $R=eRe$ is a division ring. This proves the theorem.

Corollary 1 [2] If R is a subdirectly irreducible ring having no-zero nilpotent elements and if the class of left ideals of R in H satisfies the descending chain condition, then R is a division ring.

Corollary 2 [3] If R is a subdirectly irreducible ring having no non-zero nilpotent elements and if the class of left ideals of R in H satisfies both the descending and ascending chain conditions, then R is a division ring.

Corollary 3 [4] An commutative subdirectly irreducible ring having no non-zero nilpotent elements is a field.

References

[1] Liu ShaoXue, Rings and algebras, Science Press 1983.

[2] Guo YuanChun, Acta Scientiarum Naturalium Universitatis Ji Linensis 3 (1983) 7-9.

[3] Fu ChangLin, Journal of Mathematical Research and Expositon 2 (1983) 17-22.

[4] G Birkhoff, Bulletin of the American Math Soc Vol 50 (1944) 764-768.

亚直不可约环是体的条件

王　宇

（四平师范学院数学系）

摘要　设R是结合环，H是R中全部非零理想之交，若H≠(0)，则称R是亚直不可约环。本文研究了亚直不可约环是体的条件，得到：

定理　R是具有极小单侧理想的亚直不可约环，且H中无非零幂零元，则R是体。