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The Pathwise Connected Components of Path Space on Manifold M_n*

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Let $\Omega_{A_1}^{A_2}$ be the path space consists of all piecewise C^∞-path joining point pair $A_1 \neq A_2$ on a n-dimensional connected differential manifold M_n. We know that the connectivity numbers of $\Omega_{A_1}^{A_2}$ are invariants of M_n. In other words, these quantities are independent of the point pair $A_1 \neq A_2$ on M_n. $\Omega_{A_1}^{A_2}$ is not pathwise connected in general. So we can divide $\Omega_{A_1}^{A_2}$ to several pathwise connected components. Let $C_{A_1}^{A_2}(h)$ be the pathwise connected component involving the piecewise C^∞-path h.

Theorem $A_1 \neq A_2$, $B_1 \neq B_2$ are arbitrary points on M_n. $r \in \Omega_{A_1}^{A_2}$, $k \in \Omega_{B_1}^{B_2}$ are arbitrary piecewise C^∞-path joining A_1 to A_2 and B_1 to B_2 respectively. Then $C_{A_1}^{A_2}(r) \cong C_{B_1}^{B_2}(k)$.

Corollary The connectivity numbers of $C_{A_1}^{A_2}(r)$ are invariants of M_n. Where $A_1 \neq A_2$ are arbitrary points on M_n and r is an arbitrary piecewise C^∞-path joining A_1 to A_2 on M_n.

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