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Meromorphic Functions with Two Radially Distributed Values*

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The purpose of this paper is to exhibit an extension of the results of Edrei [1] and Kimura [2], respectively. That is, we point out that the assumption of the radial distribution of poles may be removed. Thus our main results take the following forms.

Theorem 1 Let $f(z)$ be meromorphic and such that all but at most a finite number of the roots of the two equations

$$f(z)=0, \tag{1}$$

$$f^{(s)}(z)=1 \quad (s\geq 1) \tag{2}$$

be distributed on the radial $\arg z=\omega_k$ ($k=1,2,\cdots,q$; $q\geq 1$; $0\leq\omega_1<\omega_2<\cdots<\omega_q<2\pi$). Let $\rho(f)$ denote the order of $f(z)$ and

$$\beta=\max\{\frac{\pi}{\omega_2-\omega_1},\cdots,\frac{\pi}{\omega_{q+1}-\omega_q}\}(\omega_{q+1}=2\pi+\omega_1).$$

(i) If

$$\delta(0,f)+\delta(1,f^{(s)})>0, \tag{3}$$

then $\rho(f)\leq\beta$.

(ii) If the roots of one of the two equations (1) and (2) have a finite exponent of convergence ρ_1, then $\rho(f)=\rho_1$ or else $\rho_1\leq\rho(f)\leq\beta$.

Remark It is clear that the condition $s>0$ in the above theorem is necessary.

Corollary Let $f(z)$ be an entire function. If the roots of the equation

$$f'(z)+f^2(z)=1.$$

are all real, then $\rho(f)\leq 1$. If, in the above statement, the word "real" is replaced by the word "positive", then $\rho(f)\leq 1/2$.

Theorem 2 Let $f(z)$ be a meromorphic function of finite lower order and such that for any real number $\varepsilon>0$, all but at most a finite number of the roots of the equations (1) and (2) be distributed in the angles $|\arg z-\omega_k|<\varepsilon$

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$(k=1,2,\cdots,q;\ q\geqslant 1;\ 0\leqslant\omega_1<\omega_2<\cdots<\omega_q<2\pi)$.

(i) If (3) holds, then $\rho(f)\leqslant\beta$.

(ii) If the roots of one of the two equations (1) and (2) have a finite exponent of convergence ρ_1, then either $\rho(f)=\rho_1$ or else $\rho_1<\rho(f)\leqslant\beta$.

Remark Analysing the proof of the above theorems, we know that the theorems are still true, even if "(2) $f^{(s)}(z)=1$" in the theorems is replaced by "(2) $f^{(s)}(z)=p(z)$ ($p(z)$: a non-zero polynomial)" and "$\delta(1,f^{(s)})$" by "$\delta(p,f^{(s)})$".

References

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[2] W. K. Hayman, Meromorphic functions, Oxford, 1964.

[3] S. Kimura, Bull. Fac. Educ. Utsunomiya Univ, No. 24(1974), 1—5.

具有两个射线分布值的亚纯函数

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摘　　要

本文目的是把Edrei[1]和Kimura[2]中的结论进行推广，即指出即使除去极点的射线分布的假设，他们的结论仍然成立．我们主要定理是：

定理1 设 $f(z)$是亚纯的且使两方程

$$f(z)=0 \tag{1}$$

$$f^{(s)}(z)=1\quad(s\geqslant 1) \tag{2}$$

的根至多除去有限个外都位于射线组 $\arg z=\omega_k$ $(k=1,2,\cdots,q;\ q\geqslant 1;\ 0\leqslant\omega_1<\omega_2<\cdots<\omega_q<2\pi)$．设 $\rho(f)$表示 $f(z)$的级且

$$\beta=\max\left\{\frac{\pi}{\omega_2-\omega_1},\cdots,\frac{\pi}{\omega_{q+1}-\omega_q}\right\}\quad(\omega_{q+1}=2\pi+\omega_1)$$

(i) 若

$$\delta(0,f)+\delta(1,f^{(s)})>0, \tag{3}$$

则 $\rho(f)\leqslant\beta$.

(ii) 若两方程(1)和(2)之一的根有有限的收敛指数ρ_1，则 $\rho(f)=\rho_1$ 或者 $\rho_1<\rho(f)\leqslant\beta$.

同时，我们还讨论了几乎射线分布的情况．