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Second-Order Optimality Conditions for a Class of Quasidifferentiable Functions*

Tan Zhongfu

(Dalian University of Technology, Dalian)

Let $f: \mathrm{R}^n \to \mathrm{R}$ be continuously quasidifferentiable.

Definition 1 The upper and lower second-order d-directional derivatives of f at x are defined, respectively, by $f''_+(x, v^*, d) := \overline{\lim\limits_{k\to\infty}} \langle v_k + \bar{v}_k - v^*, x_k - x\rangle / t_k^2$, $f''_-(x, v^*, d) := \underline{\lim\limits_{k\to\infty}} \langle v_k + \bar{v}_k - v^*, x_k - x\rangle / t_k^2$, where (i) $t_k > 0$, $x_k \to x$; (ii) $(x_k - x)/t_k \to d$; (iii) $v_k + \bar{v}_k \to v^*$, $v_k \in \underline{\partial} f(x_k)$, $\bar{v}_k \in \bar{\partial} f(x_k)$.

We say that a continuously quasidifferentiable function f satisfies condition C^0 at x if there exists a δ-neighborhood $N_\delta(x)$ such that $\bigcup\limits_{y \in N_\delta(x)} [\underline{\partial} f(y) + \bar{\partial} f(y)]$ is bounded in a certain sense.

Theorem 1 Let x^* be a local minimizer of $f(x)$ and $d \in \mathrm{R}^n$. Suppose f satisfies the condition C^0 at x^* and $\max\{\langle v, d\rangle \mid v \in \partial_d^* f(x^*)\} = 0$. Then there exists a $v^* \in \partial_d^* f(x^*)$ satisfying $\langle v^*, d\rangle = 0$ and $f''_+(x^*, v^*, d) \geq 0$.

Theorem 2 Suppose f satisfies the condition C^0 at x^* and $d \in \mathrm{R}^n$ is an unit vector. If $\langle v, d\rangle \geq 0$, $\forall v \in \partial_d f(x^*)$ and $f''_-(x^*, 0, d) > 0$, then x^* is a local minimizer of $f(x)$.

Consider the following constrained programming

$$\text{(P)} \qquad \min\{g_0(x) \mid g_i(x) \leq 0 \ (i = 1, \cdots, m),\ g_j(x) = 0 \ (j = m+1, \cdots, q)\}.$$

where g_l $(l = 0, 1, \cdots, q)$ satisfy the condition C^0 at $x^* \in \mathrm{R}^n$.

Theorem 3 Let x^* be a local minimizer of (P) and $d \in \mathrm{R}^n$. Suppose $\max\{\langle v, d\rangle \mid v \in \partial_d^* L(x^*)\} = 0$. Then there exists a $v^* \in \partial_d^* L(x^*)$ satifying $\langle v^*, d\rangle = 0$ and $L''_+(x^*, v^*, d) \geq 0$, where $L(x) = \max\{u_0(g_0(x) - g_0(x^*)) + \sum\limits_{i=1}^{q} u_i g_i(x) \mid u_i \geq 0,\ i = 0, 1, \cdots, m,\ \sum\limits_{i=0}^{q} u_i^2 = 1\}$.

Theorem 4 Let x^* be a feasible point of (P) and d be an arbitrary unit vector in R^n. If $\langle v, d\rangle \geq 0$, $\forall v \in \partial_d \bar{L}(x^*)$ and $L''_-(x^*, 0, d) > 0$ when there exists a $v_0 \in \partial_d g_0(x^*)$ satisfying $\langle v_0, d\rangle \leq 0$, then x^* is a local minimizer of (P), where $m^* \geq 0$, $r > 0$ $\bar{L}(x) = \max\{g_0(x) - g_0(x^*) - (m^*/2)|x - x^*|^2 + r\sum\limits_{i=m+1}^{q} g_i(x), i = 1, \cdots, m\}$. (to 101)

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这与(5)矛盾．因此(u^*, λ^*, y^*)是(VD1)的有效解．

对游兆永 陈开周教授的热情指导，深表谢意！

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The Wolfe Type Duality for Nonsmooth Nonconvex Multiobjective Programming

Liu Sanyang

(Xidian University)

Abstract

In this paper, the wolfe type duality for nonsmooth nonconvex multiobjective programming is discussed using some nonconvex concepts given by author.

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