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# Hunt-Yorke's Theorem for First order Nonlinear Functional Differential Equations with Continuous Distributed Deviating Argument*

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This paper discusses the following first order functional differential equations

$$x'(t)+\int_a^b f(t,\xi,x[g(t,\xi)])\,d\sigma(\xi)=0 \qquad (b>a) \tag{1}$$

We first made the following assumptions:

($R_1$) The nonlinear function $f(t,\xi,v)$ satisfies "the bounded sublinear" condition: if $|v|\le c_0$ ($c_0>0$), then

$$|f(t,\xi,v)|\ge p(t,\xi)|v|$$

Furthermore, suppose

$$f(t,\xi,0)\equiv 0,\quad f(t,\xi,v)v>0 \quad (v\ne 0)$$

($R_2$) $\sigma:[a,b]\to R$ is a nondecreasing function and the integral in (1) is a stieltjes integral;

($R_3$) $p(t,\xi)$, $g(t,\xi)\in C[[t_0,+\infty)\times[a,b],\ R^+]$ and $g(t,\xi)\le t$, $\xi\in[a,b]$ and $\lim\limits_{t\to\infty}\min\limits_{\xi\in[a,b]} g(t,\xi)=+\infty$,

Recently, there has been a lot of activities concerning the oscillatory and asymptotic behaviors of Eq (1). See., for example [1] [2]. The important feature in this paper is the "Hunt-Yorke's Theorem" of conditions [3]. Also, we shal l establish some algebraic criterion for the existence of a nonoscillatory solution of Eq (1).

Set
$$\liminf_{t\to\infty\lambda>0}\left[\frac{1}{\lambda}\int_a^b p(t,\xi)\exp(\lambda[t-g(t,\xi)])\,d\sigma(\xi)\right]>1 \tag{2}$$

there exists $q_0, T_0\in(0,+\infty)$ such that

$$t-g(t,\xi)\le q_0 \text{ and } p(t,\xi)\le T_0 \text{ for } t\ge t_0 \text{ and } \xi\in[a,b] \tag{3}$$

there exist $\lambda_0\in(0,+\infty)$ such that

$$\overline{\lim_{t\to\infty}}\left[\frac{1}{\lambda_0}\int_a^b p(t,\xi)\exp(\lambda_0[t-g(t,\xi)])\,d\sigma(\xi)\right]<1 \tag{4}$$

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**Theorem 1** Suppose that (3) hold; then the Condition (2) implies that all solutions of (1) oscillate.

**Corollary 1** Suppose that (3) hold; then the following Condition

$$\inf_{t>t_0,\lambda>0}\left[\frac{1}{\lambda}\int_a^b p(t,\xi)\exp(\lambda[t-g(t,\xi)])\,d\sigma(\xi)\right]>1 \tag{5}$$

implies that all solutions of (1) oscillate.

**Theorem 2** Consider the linear RFDE:

$$x'(t)+\int_a^b p(t,\xi)x[g(t,\xi)]\,d\sigma(\xi)=0 \tag{H}$$

where $(R_2)$ and $(R_3)$ hold. Then (4) implies that there exists a nonoscillatory solution for Eq (H).

## References

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[2] Yuan Jiong, Acta, Math. Sinica, 30, 5, (1987), 661—670.

[3] Hunt. B. R. and Yorke J.A.J. Diff.Equs.53, 2, (1984), 139—145.

# 具有连续分布的一阶非线性泛函微分方程的Hunt-York型定理

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## 摘 要

在本文，我们建立了一类具有连续分布的非线性泛函微分方程的Hunt-York型定理；还得到了这类方程存在非振动解的代数判据.