Journal of Mathematical Research and Exposition
Vol. 11 No.4, Nov., 1991

Character and Solvability*

Li Shenglin

(Yancheng Teacher's College, Jiangsu)

As well known, the use of the character theory to study propery of groups is a major breakthrough in group theory. One of the most celebrated results is Burnside's theorem which asserts that a group with order divisible by at most two primes is solvable. This theorem uncovers that there is a close relation between the character and the solvability. It has been conjectured that only in a solvable group it is possible to have $\chi(1)^2=|G:Z(G)|$ where $\chi\in\mathrm{Irr}(G)$. In this paper, we look into the relations between character and solvability, and show that the conjecture is true under some additional conditions.

In this paper, we shall use the symbols in [1]. Let G denote always a finite group, Irr (G) be the set of all irreducible characters of G, c.d$\cdot(G)=\{\chi(1)|\chi\in\mathrm{Irr}(G)\}$. Our main results are the following

Proposition 1 If $2|\chi(1)$ for every nonlinear character $x\in\mathrm{Irr}(G)$, then G is solvable.

Proposition 2 Let $\chi\in\mathrm{Irr}(G)$. If $2\nmid\chi(1)$ and $2|\ |Z(\chi):\mathrm{Ker}\chi|$, then $G\neq G'$.

Proposition 3 Let $\chi\in\mathrm{Irr}(G)$. If $\chi(1)\neq1$ and $\chi(1)^2=|G:Z(G)|$, then $|Z(G):\mathrm{Ker}x|>1$.

Proposition 4 Let $\chi\in\mathrm{Irr}(G)$, $\chi(1)\neq1$. If $\chi(1)^2=|G:Z(G)|$ and $|Z(G):\mathrm{Ker}\chi|>2$, then G has at least one irreducible character χ_1 such that $\chi_1\neq\chi$, $\mathrm{Ker}\chi_1=\mathrm{Ker}\chi$ and $\chi_1(1)=\chi(1)$.

Proposition 5 Let $|\mathrm{Irr}(G)|<4$, and there is an irreducibe charactor $\chi\in\mathrm{Irr}(G)$ with $\chi(1)^2=|G:Z(G)|$. Then G is solvable.

Proposition 6 Let $|\mathrm{Irr}(G)|=5$, and there is isan irreducible charactor $\chi\in\mathrm{Irr}(G)$ with $\chi(1)^2=|G:Z(G)|$. If $3\nmid\chi(1)$, then G is solvable.

Proposition 7 Let $|\mathrm{Irr}(G)|=6$, and there is an irreducible character $\chi\in\mathrm{Irr}(G)$ with $\chi(1)^2=|G:Z(G)|$, $3\nmid\chi(1)$ and $|Z(G):\mathrm{Ker}(\chi)\nmid>2$. Then G is solvable.

Proposition 8 Let $|\mathrm{Irr}(G)|\leq6$, and there is an irredicible character $\chi\in\mathrm{Irr}(G)$ with $\chi(1)^2=|G:Z(G)|$. Assume either $\chi_1(1)=\chi_2(1)$ or $3\nmid(\chi_1(1),\chi_2(1))$, for all

* Received Feb. 18, 1990.

χ_1, $\chi_2 \in \mathrm{Irr}(G)$. Then G is solvable.

Proposition 9 Let $|\mathrm{c.d.}(G)| = 4$, and there is an irreducible character $\chi \in \mathrm{Irr}(G)$ with $\chi(1)^2 = |G:Z(G)|$. Then $G \neq G'$.

References

[1] I. M. Lsaacs, Character Theory of Finite Groups, Academic Press, New York 1976.

特 征 与 可 解 性

李 胜 林

（盐城师范专科学校，江苏）

本文研究了具有不可约特征x使得$x(1)^2 = |G: Z(G)|$成立的群G的可解性，并证明了：对于这样的群G，当$|\mathrm{Irr}(G)| \leq 4$时，G是可解群；当$4 < |\mathrm{Irr}(G)| \leq 6$时，在某些条件下，$G$是有解群.