Journal of Mathematical Research & Exposition,
Vol.12, No.2, May. 1992

Improvement of Sufficient Conditions for Stability of Discrete Scheme*

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A.A. Samarsky has given some sufficient conditions for stability. For the norm $\|\phi\|$, the two layer scheme require $B \geq \varepsilon E + o.5\tau A$, the three layer scheme require $B \geq \varepsilon E$, where $\varepsilon(>0)$ does not depend on τ and h. In this paper for lower condition $B > 0.5\tau A, B > 0$, we still obtain the correspoundent theorems . we use the definition and notation in [1].

1. Sufficient Conditions of Stability for Two Layer Scheme

Theorem 1.1 *Assume the scheme*

$$By_t + Ay = \phi \qquad , y(0) = y_0 \tag{1}$$

be original scheme variety. If

$$B > 0.5\tau A, \tag{2}$$

then scheme (1) is stable and the solution of problem (1) has priori estimation

$$\|y(t+\tau)\|_{\bar{A}} \leq \|y_0\|_{\bar{A}} + \frac{1}{\sqrt{2}}(\sum_{t'=0}^{t} \tau\|\phi(t')\|^2)^{\frac{1}{2}}, \tag{3}$$

where $\bar{A} = \varepsilon(\tau, h)A, \varepsilon(\tau, h) > 0$.

Theorom 1.2 *Assume* $A = A^* > 0$ *and*

$$\sigma > \sigma_0, \sigma_0 = \frac{1}{2} - \frac{1}{\tau\|A\|}, \tag{4}$$

then scheme

$$y_t + A(\sigma\hat{Y} + (1-\sigma)Y) = \phi, y(0) = y_0 \tag{5}$$

is stable and has priori estimation (3).

Theorem 1.3 *Assume* $A(t) = A^*(t) > 0$ *and satisfy condition (4), them scheme (5) is stable and has estimation*

$$\|y(t+\tau)\|_{\bar{A}} \leq \|y_0\|_{\bar{A}} + \frac{1}{\sqrt{2}}(\sum_{t'=0}^{t} \tau\|A^{-1}(t')\phi(t')\|^2)^{\frac{1}{2}}, \tag{6}$$

$$\|y(t+\tau)\|_{\bar{\bar{A}}} \leq \|y_0\|_{\bar{\bar{A}}} + \frac{1}{\sqrt{2}}(\sum_{t'=0}^{t} \tau\|\phi(t')\|^2{}_{A^{-1}})^{\frac{1}{2}} \tag{7}$$

*Received March 8, 1990.

where $\bar{A} = \varepsilon(\tau, h, , t)E, \bar{\bar{A}} = \varepsilon(\tau, h, t)\delta(\tau, h, t)E,\ \varepsilon(\tau, h, t) > 0, \delta(\tau, h, t) > 0,$

Theorem 1.4 *Assume*

$$\begin{cases} A(t) = A^*(t) > 0, for\ all\ t \in \bar{\omega}_\tau, \\ A(t) is\ Lipschitz\ continuous \\ B(t) > 0, for\ all\ t \in \bar{\omega}_\tau \end{cases} \tag{8}$$

and

$$B(t) > \varepsilon(\tau, h)E + o.5\tau A(t), for\ all\ t \in \omega_\tau, \tag{9}$$

then the scheme (1) is stable and has priori estimation

$$\|y(t+\tau)\|_{\bar{A}(t)} \leq M_1(\|y(0)\|_{\bar{A}(0)} + \frac{1}{\sqrt{2}}(\sum_{t'=0}^{t} \tau\|\phi(t')\|^2)^{\frac{1}{2}}, \tag{10}$$

where $M_1 = e^{0.5c_3t_0}, \bar{A}(t) = \varepsilon(\tau, h)A(t)\ \varepsilon(\tau, h) > 0.$

Corollary 1.1 *If the elements* $b_{ij}(t) - 0.5\tau a_{ij}(t)(i, j = 1, 2, ..., N)$ *in* $B(t) - 0.5\tau A(t)$ *are continuous or only have finite jump point of the first kind, then substituting condition* $B(t) > 0.5\tau A(t)$ *for condition(9), the theorem 1.4 still holds.*

2. Sufficient Conditions of Stability for Three Layer Scheme

Theorem 2.1 For scheme

$$By_{\hat{t}} + \tau^2 RY_{\bar{t}t} + AY = \phi, Y(\tau) = Y_1, Y(0) = Y_0, \tag{11}$$

assume $A = A^* > 0, R = R^* > 0$, **them under conditions** $B > 0, R \geq \frac{1}{4}A$, **the scheme(11)is stable and its solution has priori estimation**

$$\|Y(t+\tau)\| \leq \|y(\tau)\| + \frac{1}{\sqrt{2}}(\sum_{t'=\tau}^{t} \tau\|\phi(t')\|^2)^{\frac{1}{2}}, \tag{12}$$

where $\|Y(t+\tau)\| = \frac{1}{4}(\bar{A}(Y(t) + Y(t+\tau)),\ y(t) + y(t+\tau)) + \tau^2((\bar{R} - \frac{1}{4}\bar{A})y_t(t), y_t(t)),$ $\bar{R} = \varepsilon(\tau, h)R, \bar{A} = \varepsilon(\tau, h)A,\ \varepsilon(\tau, h,) > 0.$

Theorem 2.2 Assume $A(t) = A^*(t) > 0, R(t) = R^*(t) > 0, B(t) \geq \bar{\varepsilon}(\tau, h)E, A(t), R(t) - \frac{1}{4}A(t)$ are Lipschitz continuous and

$$R(t) \geq \frac{1}{4}A(t), for\ all\ 0 < t = n\tau < t_0, \tag{13}$$

then the scheme (11)is stable and has estimation

$$\|Y(t+\tau)\|_{(t)} \leq M_1\|Y(\tau)\|_{(0)} + M_2(\sum_{t'=\tau}^{t} \tau\|\phi(t')\|^2)^{\frac{1}{2}}, \tag{14}$$

where

$$Y(t+\tau)\|_{(t)} = \frac{1}{4}(\bar{A}(t)(y(t)+y(t+\tau)), y(t)+y(t+\tau)) + \tau^2((\bar{R}(t) - \frac{1}{4}\bar{A}(t))y_t(t), y_t(t)), \quad (15)$$

$M_1 = e^{0.5c_3t_0}$, $M_2 = \frac{1}{\sqrt{2}}$, c_3 *is a positive constant,* $\bar{\varepsilon}(\tau,h) > 0$.

Theorem 2.3 Assume $A(t) = A^*(t) > 0, R(t) = R^*(t) > 0$, $B(t) \geq \bar{\varepsilon}(\tau,h)E, A(t), R(t)$ Lipschitz continuous and

$$R(t) \geq \frac{1+\varepsilon}{4}A(t), \textit{for all } 0 \leq t = n\tau \leq t_0, \quad (16)$$

where $\varepsilon(>0)$ does not depend on τ and h, then scheme (11) is stable and has estimation

$$\|Y(t+\tau)\|_{(t)} \leq M_1\|Y(\tau)\|_{(0)} + M_2(\sum_{t'=\tau}^{t} \tau\|\phi(t')\|^2)^{\frac{1}{2}}, \quad (17)$$

where $\bar{\varepsilon}(\tau,h) > 0$, $M_1 = e^{0.5\frac{2+\varepsilon}{\varepsilon}c_3t_0}$, $M_2 = \frac{1}{\sqrt{2}}$.

Corollary 2.1 If the elements $b_{ij}(t)(i,j = 1,2...,N)$ in $B(t)$ are continuous or only have finite jump point of the first kind. then substituting condition $B(t) > 0$ for condition $B(t) \geq \bar{\varepsilon}(\tau,h)E$, Theorem 2.2, 2.3 still hold.

REFERENCE

*[1] A.A.Samarsky,*Introduction to the Analysis of Difference Schenes, *Moscow:Nauka, 1971(In Russian).*

离散格式稳定的充分条件的改进

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摘　要

本文改进了 $A.A.$ 萨多尔斯劳[1]中判稳的充分条件,把条件 $B\geqslant\varepsilon E+0.5\tau A, B\geqslant\varepsilon E$ 放宽为:基本上只要求 $B>0.5\tau A, B>0$.