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On Compact Submanifolds in a Sphere*

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Abstract. In this paper the Pinching theory for compact submanifolds in a unit sphere is considered. We improve the Pinching constants of S.T. Yau and X.H. Mo for compact submanifolds with parallel mean curvature in sphere and those of S.S. Chern and Y.B. Shen for compact minimal submanifolds. For sectional curvature, we improve the Pinching constant of Y.B. Shen.

Let M^n be an $n-$dimensional compact submanifold in a unit sphere S^{n+p}, S denotes the squere of the length of the second fundamental form of the immersion: $M^n \hookrightarrow S^{n+p}$. In [1], S.T. Yau proved that if the mean curvature is parallel, and S is not greater than

$$n/(\sqrt{n}+3-\frac{1}{p-1}), \tag{1}$$

then M^n lies in a totally geodesic S^{n+1}.

On the other hand, if M^n is minimal, and

$$S \leq n/(2-\frac{1}{p}), \tag{2}$$

then M^n is totally geodesic, Clifford torus or Veronese surface in S^4 (refer to S.S. Chern, M. Do Carmo, S. Kobayashi [2]).

The two kinds of Pinching constant are improved in this paper. We have

Theorem 1 *Let M^n be an $n-$dimensional compact submanifold with parallel mean curvature in S^{n+p} with $p \geq 2, n \geq 2, H \neq 0$. If*

$$S \leq \min\{n/(2-\frac{1}{p-1}), n/\sqrt{\frac{n+3}{2}}\}, \tag{3}$$

then M^n lies in a totally geodesic S^{n+1}.

Theorem 2 *Let M^n be an $n-$dimensional compact minimal submanifold, satisfying*

$$S \leq \frac{3n+2}{5n+2}n, \tag{4}$$

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then we have either

(1) M^n is totally geodesic in S^{n+p};

or

(2) $n=2, M^2$ is the Veronese surface in S^4.

In addition to that, we consider the Pinching theory for sectional curvature, and we obtain

Theorem 3 *Let M^n be an $n-$dimensional compact minimal submanifold with constant scalar curvature, if the sectional curvature of M^n is nowhere less than $C_S = 1/2-(3n+2)/(2p(5n+2))$, then M^n is totally geodesic or the Veronese surface in S^4.*

Remark 1 Condition (3) is the improvement of (1) for the Pinching constant. And the Pinching constant in [4] is $\max\{\frac{n(1+H^2)}{\sqrt{n-1}+1}, \frac{n(1+2H^2)}{\sqrt{n}+1}\}$, in fact,

$$H^2 = \frac{1}{n^2}\sum_\alpha(\sum_i H_{ii}^\alpha)^2 \le \frac{1}{n}\sum_{\alpha,i}(h_{ii}^\alpha)^2 \le \frac{1}{n}S,$$

it follows that

$$H^2 \le \max\{\frac{1}{\sqrt{n-1}}, \frac{1}{\sqrt{n}-1}\},$$

taking this inequality into the preceding formula, we have

$$S \le \max\{\frac{n}{\sqrt{n-1}}, \frac{n}{\sqrt{n}-1}\}.$$

So the Pinching constant in this paper is better than that in [4] when the dimension n is a little larger.

Remark 2 Condition (4) is better than (2) under the assumption $p\ge 3$. In fact, the costant $[(3n+2)/(5n+2)]n$ in (4) is also better than the constant $n/(1+\sqrt{\frac{n-1}{2n}})$ in [3].

Remark 3 The constant C_S in Theorem 3 is better than the corresponding constnat $C_S = 1/2-(2n\sqrt{2n(n-1)})/(2p(n+1)$ [3] when p and n satisfy the following inequality

$$\frac{3n+2}{n+2} \le p \le \frac{(3n+2)(n+1)}{5n+2},$$

then

$$C_S \le \min\{\frac{p-1}{2p-1}, \frac{n}{2(n+1)}\}.$$

Hence we partially improve the results of S.T. Yau [1] and T. Itoh [5].

We shall give the proof to these results in the next sections.

§1. Decreasing of the Codimension

Let M^n be an $n-$dimensional Riemannian manifold immersed isometrically is an $(n+p)-$dimensional unit sphere S^{n+p} $(p\ge 2)$, we choose a local field of orthonormal frames $e_1, e_2, \cdots, e_{n+p}$ in S^{n+p} such that restricted to M^N the vectors $e_1, e_2, \cdots, e_n$ are tangent to M^n. We shall make use of the following convention on the ranges of indices: $1 \leq A, B, C, \cdots \leq n+p, 1 \leq i, j, k \cdots \leq n, n+1 \leq \alpha, \beta, \gamma, \cdots \leq n+p$, and we ahall agree that repeated indices are summed over the respective ranges. With respect to the frame field of S^{n+p} chosen above, let $\omega_1, \cdots, \omega_{n+p}$ be the field of dual frames. Then the structure equations of S^{n+p} are given by [1]

$$d\omega_A = -\sum_B \omega_{AB} \wedge \omega_B, \tag{1.1}$$

$$\omega_{AB} + \omega_{BA} = 0,$$
$$d\omega_{AB} = -\sum_C \omega_{AC} \wedge \omega_{CB} + \phi_{AB}, \tag{1.2}$$

$$\phi_{AB} = \frac{1}{2} \sum_{C,D} K_{ABCD} \omega_C \wedge \omega_D,$$

$$K_{ABCD} = \delta_{AC}\delta_{BD} - \delta_{AD}\delta_{BC}. \tag{1.3}$$

We restrict these forms to M^n, then

$$\omega_{\alpha i} = \sum_j h_{ij}^\alpha \omega_j, h_{ij}^\alpha = h_{ji}^\alpha, \tag{1.4}$$

$$\omega_\alpha = 0, \tag{1.5}$$

$$d\omega_{ij} = -\sum_k \omega_{ik} \wedge \omega_{kj} + \Omega_{ij}, \tag{1.6}$$

$$\Omega_{ij} = \frac{1}{2} \sum_{k,l} R_{ijkl} \omega_k \wedge \omega_l, \tag{1.7}$$

$$R_{ijkl} = K_{ijkl} + \sum_\alpha (h_{ik}^\alpha h_{jl}^\alpha - h_{il}^\alpha h_{jk}^\alpha), \tag{1.8}$$

$$d\omega_{\alpha\beta} = -\sum_r \omega_{\alpha r} \wedge \omega_{r\beta} + \Omega_{\alpha\beta},$$

$$\Omega_{\alpha\beta} = \frac{1}{2} \sum_{k,l} R_{\alpha\beta kl} \omega_k \wedge_l, \tag{1.9}$$

$$R_{\alpha\beta kl} = \sum_i (h_{ik}^\alpha h_{il}^\beta - h_{il}^\alpha h_{ik}^\beta). \tag{1.10}$$

We denote by $B = \sum_{\alpha,i,j} h_{ij}^\alpha \omega_i \omega_j e_\alpha$ the second fundamental form of M^n and $S = \sum_{\alpha,i,j} (h_{ij}^\alpha)^2$ the length square of B. We denote by H_α the matrix (h_{ij}^α) for every α. We call $\xi = \frac{1}{n} \sum_\alpha \operatorname{tr}(H_\alpha) e_\alpha$ the mean curvature vector, the length of it $H = \| \xi \|$ is called the length of the mean curvature.

If the mean curvature vector ξ is parallel in the normal boundle, it is easy to know $H =$ constant. Suppose $H \neq 0$, let $e_{n+1} = \xi / H$, therefore we have

$$\operatorname{tr} H_\alpha = 0 \quad (\alpha \neq n+1), \operatorname{tr} H_{n+1} = nH, \tag{1.11}$$

$$\omega_{n+1,\alpha} = 0, \tag{1.12}$$

$$H_{n+1}H_\alpha = H_\alpha H_{n+1}, \quad \text{for every} \quad \alpha. \tag{1.13}$$

The proof of Theorem 1 is indeced from the inequality below.

Lemma *If $a_1, a_2, \cdots, a_n; b_1, b_2, \cdots, b_n$ are $2n$ real numbers, satisfying $\sum_{i=1}^n b_i = 0$, then we have*

$$\left(\sum_{i,j} a_i a_j (b_i - b_j)^2\right)^2 \le (2n+6)\left(\sum_i a_i^2\right)^2 \left(\sum_j b_j^2\right)^2. \tag{1.14}$$

Proof By Schwarz inequality, we have

$$\begin{aligned}
&\left(\sum_{i,j} a_i a_j (b_i - b_j)^2\right)^2 \le (\sum_i a_i^2)^2 \sum_{i,j} (b_i - b_j)^4 \\
= &\ (\sum_i a_i^2)^2 \sum_{i,j} (b_i^4 + b_j^4 + 6b_i^2 b_j^2 - 4b_i b_j^3 - 4b_i^3 b_j) \\
= &\ (\sum_i a_i^2)^2 [2n \sum_i b_i^4 + 6(\sum_j b_j^2)^2] \le (2n+6)(\sum_i a_i^2)^2 (\sum_j b_j^2)^2.
\end{aligned}$$

This completes the proof the the lemma.

Next, we give the proof of Theorem 1.

From (2.3) in [4] (refer to (7.8)–(7.12) in [1]), one can easily see that

$$\begin{aligned}
\sum_{\beta \ne n+1} h_{ij}^\beta \Delta h_{ij}^\beta \ \ge \ & nH \sum_{\beta \ne n+1} \mathrm{tr}(H_{n+1} H_\beta^2) - \sum_{\beta \ne n+1} [\mathrm{tr}(H_{n+1} H_\beta)]^2 \\
& + n \sum_{\beta \ne n+1} (h_{ij}^\beta)^2 - (2 - \frac{1}{p-1})[\sum_{\beta \ne n+1} (h_{ij}^\beta)^2]^2.
\end{aligned} \tag{1.15}$$

Now fix a vector e_β ($\beta \ne n+1$), from (1.11) and (1.13), let H_{n+1} and H_β be diagonalized simultaneously, then we have

$$\begin{aligned}
& nH \mathrm{tr}(H_{n+1} H_\beta^2) - [\mathrm{tr}(H_{n+1} H_\beta)]^2 \\
= & \sum_{i,j} h_{ii}^{n+1} h_{jj}^{n+1} (h_{jj}^\beta)^2 - (\sum_i h_{ii}^{n+1} h_{ii}^\beta)^2 \\
= & \sum_{i,j} [h_{ii}^{n+1} h_{jj}^{n+1} (h_{jj}^\beta)^2 - h_{ii}^{n+1} h_{ii}^\beta h_{jj}^{n+1} h_{jj}^\beta] \\
= & \frac{1}{2} \sum_{i,j} h_{ii}^{n+1} h_{jj}^{n+1} (h_{ii}^\beta - h_{jj}^\beta)^2.
\end{aligned} \tag{1.16}$$

Notice that $\mathrm{tr} H_\beta = \sum_i h_{ii}^\beta = 0$, from Lemma above, we have

$$\begin{aligned} & nH\mathrm{tr}(H_{n+1}H_\beta^2) - [\mathrm{tr}(H_{n+1}H_\beta)]^2 \\ \geq\ & -\frac{\sqrt{2n+6}}{2}(\sum_i (h_{ii}^{n+1})^2(\sum_j (h_{jj}^\beta)^2) \\ =\ & -\sqrt{\frac{n+3}{2}}(\sum_{i,j}(h_{ij}^{n+1})^2)(\sum_{i,j}(h_{ij}^\beta)^2). \end{aligned} \tag{1.17}$$

Substituting (1.17) into (1.15), we get

$$\begin{aligned} \sum_{\beta\neq n+1} h_{ij}^\beta \Delta h_{ij}^\beta \ \geq\ & \sum_{\beta\neq n+1}(h_{ij}^\beta)^2\{n - \sqrt{\frac{n+3}{2}}\sum_{i,j}(h_{ij}^{n+1})^2 - (2-\frac{1}{p-1})\sum_{\substack{\beta\neq n+1\\ i,j}}(h_{ij}^\beta)^2\} \\ \geq\ & \sum_{\substack{\beta\neq n+1\\ i,j}}(h_{ij}^\beta)^2(n-MS), \end{aligned} \tag{1.18}$$

where $M = \max\{2-\frac{1}{p-1}, \sqrt{\frac{n-3}{2}}\}$. When $S \leq n/M$,

$$\frac{1}{2}\Delta\sum_{\substack{\beta\neq n+1\\ i,j}}(h_{ij}^\beta)^2 = \sum_{\substack{\beta\neq n+1\\ i,j,k}}(h_{ijk}^\beta)^2 + \sum_{\substack{\beta\neq n+1\\ i,j}} h_{ij}^\beta\Delta h_{ij}^\beta \geq 0, \tag{1.19}$$

from which, it follows that $\sum_{\substack{\beta\neq n+1\\ i,j}}(h_{ij}^\beta)^2$ is constant. Then (1.19) becomes equality, hence $0 = \sum_{\substack{\beta\neq n+1\\ i,j}} h_{ij}^\beta\Delta h_{ij}^\beta \geq \sum_{\substack{\beta\neq n+1\\ i,j}}(h_{ij}^\beta)^2(n-MS) \geq 0$ and (1.18) becomes equality. So it is easy to see

(1). When $\sqrt{\frac{n+3}{2}} > 2 - \frac{1}{p-1}$, we have

$$\sum_{\beta\neq n+1}(h_{ij}^\beta)^2 = 0.$$

Therefore, M^n lies in a totally geodesic S^{n+1};

(2). When $\sqrt{\frac{n+3}{2}} \leq 2-\frac{1}{p-1}$, obviously $2 \leq n \leq 4$, and $\sqrt{\frac{n+3}{2}} \neq 2 - \frac{1}{p-1}$, hence $\sqrt{\frac{n+3}{2}} < 2 - \frac{1}{p-1}$, from which we obtain $\sum_{i,j}(h_{ij}^{n+1})^2 = 0$ and $H = 0$, which contradicts the assumption $H \neq 0$.

This completes the proof of Theorem 1.

Corollary *Let $n \geq 5$ and the conditions in Theorem 1 are satisfied. Then M^n is a totally umbilical submanifold in S^{n+1}.*

Proof From the Corollary of §2 in [4] and the fact $\sqrt{\frac{n+3}{2}} \geq 2 - \frac{1}{p-1}$, when $n \geq 5$, it is straightforward to see

$$n/\sqrt{\frac{n+3}{2}} \leq 2\sqrt{n-1},$$

so we have the result at once. This completes the proof of the corollary.

§2. On Minimal Submanifold

In this section we make use of the frame field in [6].

Suppose M^n is minimal, i.e., $H=0$. If $\cup M$ is the unit tangent boundle over M^n, i.e., $\cup M=\cup_{x\in M^n}\cup M_x$, where $\cup M_x=\{u\in TM_x, \| u \|=1\}$. We define a function

$$\sigma:\cup M\longrightarrow R,$$
$$\sigma(u)=\langle B(u,u),B(u,u)\rangle,\forall u\in\cup M.(21.)$$

Since $\cup M$ is compact, σ attains its maximum at a vector in $\cup M$. Suppose that this vector is $u_0\in\cup M_x$ for some point $x_0\in M^n$. If $\sigma(u_0)=0$, obviously, M^n is totally geodesic. So we let $\sigma(u_0)\neq 0$ in the following part.

The convention on the ranges of indices is the same that in §1. In the neighbourhood of x_0, we choose a local field of orthonormal frames as in §1. By taking $u_0=e_1$ and letting

$$B(u_0,u_0)/\parallel B(u_0,u_0)\parallel=e_{n+1}, \tag{2.2}$$

at point x_0, we have

$$h_{11}^{\alpha}=0,\quad \alpha\neq n+1. \tag{2.3}$$

Since e_1 is a maximal direction, at the point x_0 for any $t,x^2,\cdots,x^n\in\mathbf{R}$, we have

$$\sigma(e_1+t\sum_{k=2}^{n}x^ke_k)\leq[1+t^2\sum_{k=2}^{n}(x^k)^2]^2(h_{11}^{n+1})^2. \tag{2.4}$$

Expanding (2.4) in terms of t, we obtain

$$4th_{11}^{n+1}\sum_{k=2}^{n}x^kh_{1k}^{n+1}+O(t^2)\leq 0.$$

In follows that

$$h_{1k}^{n+1}=0\ (k\neq 1) \tag{2.5}$$

at point x_0.

We now choose a proper frame $e_2,\cdots,e_n$ at $x_0\in M$ such that H_{n+1} is diagonalized, i.e.,

$$h_{ij}^{n+1}=0\quad(i\neq j). \tag{2.6}$$

Once more expanding (2.4) in terms of t, we obtain

$$-2t^2\{\sum_{i\neq 1}[h_{11}^{n+1}(h_{11}^{n+1}-h_{ii}^{n+1})-2\sum_{\alpha\neq n+1}(h_{1i}^{\alpha})^2](x^i)^2-4\sum_{\alpha\neq n+1}\sum_{\substack{i,j\neq 1\\ i\neq j}}h_{1i}^{\alpha}h_{1j}^{\alpha}x^ix^j\}+O(t^3)\leq 0.$$

It follows that

$$2\sum_{\alpha\neq n+1}(h_{1i}^{\alpha})^2\leq h_{11}^{n+1}-h_{ii}^{n+1}) \tag{2.7}$$

at point x_0.

In addition to that, from (2.3) above and (1.13) in [3], i.e.,

$$\sum_\alpha (h^\alpha_{11i})^2 + \sum_\alpha h^\alpha_{11} h^\alpha_{11ii} \leq 0,$$

we get

$$h^{n+1}_{11} h^{n+1}_{11ii} \leq 0 \tag{2.8}$$

at point x_0.

Now we give the proof of Theorem 2. All calculation is restricted at point x_0.

Suppose M^n is not totally geodesic from the convention of (2.2), we have (2.3)–(2.8). For $u = \sum_i u^i e_i \in \cup M$, since

$$\sigma(e_1) = (h^{n+1}_{11})^2 = \max_{u \in \cup M} \sigma(u) = \max \sum_\alpha (\sum_{i,j} h^\alpha_{ij} u^i u^j)^2 \neq 0, \tag{2.9}$$

it is straightforward to see

$$|h^{n+1}_{11}| = \max_i \{|h^{n+1}_{ii}|\}. \tag{2.10}$$

From (2.7) summing by index i, we get

$$2 \sum_{\substack{\alpha \neq n+1 \\ i}} (h^\alpha_{1i})^2 \leq n\sigma(e_1). \tag{2.11}$$

On the other hand, from (2.7) and (2.10), one can obtain

$$\sum_{\substack{\alpha \neq n+1 \\ i}} (h^\alpha_{1i})^2 \leq (h^{n+1}_{11})^2 = \sigma(e_1). \tag{2.12}$$

Making use of Ricci identical relation and (2.6), from (2.8) summing by index i, we have

$$\begin{aligned} 0 &\geq \sum_i h^{n+1}_{11}(h^{n+1}_{ii} R_{i11i} + h^{n+1}_{11} R_{1i1i}) + h^{n+1}_{11} \sum_{\alpha,i} h^\alpha_{1i} R_{\alpha,n+1,1i} \quad (2.13) \\ &= n\sigma(e_1) - 2\sigma(e_1) \sum_{\substack{\alpha \neq n+1 \\ i}} (h^\alpha_{1i})^2 - \sigma(e_1) \sum_i (h^{n+1}_{ii})^2 + 2 \sum_{\substack{\alpha \neq n+1 \\ i}} h^{n+1}_{11} h^{n+1}_{ii} (h^\alpha_{1i})^2. \end{aligned}$$

Substituting $2h^{n+1}_{11} h^{n+1}_{ii} \geq -(h^{n+1}_{11})^2 - (h^{n+1}_{ii})^2$ into (2.13), we can obtain

$$0 \geq n\sigma(e_1) - 3\sigma(e_1) \sum_{\substack{\alpha \neq n+1 \\ i}} (h^\alpha_{1i})^2 - \sigma(e_1) \sum_i (h^{n+1}_{ii})^2 - (\lambda + \mu) \sum_{\substack{\alpha \neq n+1 \\ i}} (h^{n+1}_{ii})^2 (h^\alpha_{1i})^2, \tag{2.14}$$

where $\lambda \geq 0, \mu \geq 0, \lambda + \mu = 1$. By (2.3), (2.10), (2.12), we have

$$\begin{aligned} 0 &\geq n\sigma(e_1) - (3+\lambda)\sigma(e_1) \sum_{\substack{\alpha \neq n+1 \\ i}} (h^\alpha_{1i})^2 - \sigma(e_1) \sum_i (h^{n+1}_{ii})^2 - \mu\sigma(e_1) \sum_{i \neq 1} (h^{n+1}_{ii})^2 \\ &= n\sigma(e_1) - (3+\lambda)\sigma(e_1) \sum_{\substack{\alpha \neq n+1 \\ i}} (h^\alpha_{1i})^2 - (1+\mu)\sigma(e_1) \sum_i (h^{n+1}_{ii})^2 + \mu\sigma^2(e_1). \quad (2.15) \end{aligned}$$

Together with (2.11), (2.15) becomes

$$0 \geq n\sigma(e_1) - (3+\lambda-\frac{2\mu}{n})\sigma(e_1)\sum_{\substack{\alpha\neq n+1\\ i}}(h_{1i}^{\alpha})^2 - (1+\mu)\sigma(e_1)\sum_i(h_{ii}^{n+1})^2. \tag{2.16}$$

Take $\lambda = (n+2)/(3n+2)$, therefore $1+\mu = (5n+2)/(3n+2), 3+\lambda-2\mu/n = (10n+4)/(3n+2)$, substituting them into (2.16), we have

$$0 \geq \sigma(e_1)\{n - \frac{5n+2}{3n+2}[2\sum_{\substack{\alpha\neq n+1\\ i}}(h_{1i}^{\alpha})^2 + \sum_i(h_{ii}^{n+1})^2]\} \geq \sigma(e_1)(n - \frac{5n+2}{3n+2}S) \geq 0.$$

From $\sigma(e_1) \neq 0$, it follows that $S = n(3n+2)/(5n+2)$. So the previous inequalitites become equalities. Hence $|h_{ii}^{n+1}| = |h_{11}^{n+1}|$ $(i \neq 1)$. By (1.12), we know

$$\sum_{\alpha\neq n+1}(h_{1i}^{\alpha})^2 = (h_{11}^{n+1})^2, \tag{2.17}$$

Substituting (2.17) into (2.7), one can easily see

$$h_{ii}^{n+1} = -h_{11}^{n+1}, i \neq 1. \tag{2.18}$$

But $\sum_i h_{ii}^{n+1} = 0$, therefore $n = 2$.

Then the condition of the theorem becomes $S \leq 4/3$, i.e., the Gauss curvature K of M^2 is not less than 1/3. From the well-known result of [5], we obtain that M^2 is the Veronese surface in S^4.

This completes the proof of Theorem 2.

Making use of the result above, we can prove Theorem 3 as follows: From the integration inequality (10.1) in [1], we obtain straightly

$$0 \geq \int_M S(2nK_M + \frac{1}{p}S - n) * 1, \tag{2.19}$$

where K_M denotes the infinium of the sectional curvature of M^n. From the assumption of the theorem, we see S =constant. Suppose

$$S > \frac{3n+2}{5n+2}n, \tag{2.20}$$

substituting (2.20) into (2.19), we have

$$0 \geq \int_M S\{K_M - (\frac{1}{2} - \frac{S}{2pn})\} * 1 > \int_M S\{K_M - (\frac{1}{2} - \frac{3n+2}{2p(5n+2)})\} * 1 \geq 0. \tag{2.21}$$

This contradiction shows that (2.20) is also not true. Hence $S \leq n(3n+2)/(5n+2)$. Theorem 3 follows directly form Theorem 2.

This completes the proof Theorem 3.

On the other hand, from the proof of Theorem 2, it is easy to get

Corollary *Let M^n be an $n-$dimensional compact minimal submanifold in S^{n+p}, if $n \geq 3$ and $S(x_0) \leq n(3n+2)/(5n+2)$, then M^n is totally geodesic, where x_0 is the special point in the proof of Theorem 2.*

Proof Suppose M^n is not totally geodesic, i.e., $\sigma(e_1) \neq 0$. We have

$$0 \geq \sigma(e_1)\{n - \frac{5n+2}{3n+2}S(x_0)\}.$$

Introducing $S(x) \leq (3n+2)/(5n+2)$ into above inequality, we get

$$S(x_0) = \frac{3n+2}{5n+2}n.$$

In the same way as in the proof of Theorem 2, we obtain $h_{ii}^{n+1} = -h_{11}^{n+1}$ $(i \neq 1)$, but $\sum_i h_{ii}^{n+1} = 0$, it follows that $n = 2$, it contradicts the fact $n \geq 3$.

This completes the proof of the Corollary.

References

[1] S.T. Yau, *Submanifolds with constant mean curvature, I*, Amer. J. Math., **96**(1974), 346–366, **II.**, **97**(1975), 76–100.

[2] S.S. Chern, M. Do Carmo and S.Kobayashi,*Minimal submanifolds of a sphere with second fundamental form of constant length*, in Shiing-Shen Chern Selected Papers, 393–409, Springer-Verlag, 1978.

[3] Shen Y.B.,*On intrinsic rigidity for minimal submanifolds in a sphere*, Acta Sinica, (Ser A), **32**(1989), 769–781.

[4] Mo X.H., *Submanifolds with parallel mean curvature in a Riemanniam manifold with constant sectional curvature*, Chinese Ann. of Math., **9**A(1988), 530–540.

[5] T. Itoh,*Addendum to my paper "On Veronese manifolds"*, J. Math. Soc. Japan, **30**(1978), 73–74.

[6] H. Cauchman,*Minimal submanifolds of a sphere with Rounded second fundamental form*, Tran. Amer. Math. Soc.,**298**(1986), 779- -791.

关于球面的紧子流形

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摘　要

本文讨论单位球面的紧致子流形的 Pinching 问题. 改进了 S. T. Yau 与莫小欢有关球面中具有平行平均曲率的紧致子流形的 Pinching 常数及 S. S. Chern 等与沈一兵有关紧致极小子流形的 Pinching 常数. 对截曲率的情形,本文还改进了沈一兵有关截曲率的 Pinching 常数.