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A Special Decomposition for Symmetric Matrix*

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Theorem *Suppose $K=(k_{ij})_{n\times n}$ is a symmetrical matrix, then K can be decomposed $K=-AHA^T$, where A contains only elements 0, 1 and -1; H is a diagonal matrix.*

Proof Let $A=(A^{(1)},A^{(2)},\cdots,A^{(n)})\in R^{n\times\frac{n(n+1)}{2}}$, where $A^{(j)}\in R^{n\times(n-j+1)}$ $(j=1,2,\ldots,n)$. Let

$$H=\mathrm{diag}(H^{(1)},H^{(2)},\cdots,H^{(m)})\in R^{\frac{n(n+1)}{2}\times\frac{n(n+1)}{2}},$$

where $H^{(j)}\in R^{(n-j+1)\times(n-j+1)}$ with a diagonal form. Denote the elements of $A^{(j)}$ and $H^{(j)}$ as follows:

$$\begin{cases} a_{j,i}^{(j)}=1, & \text{when } i=1,2,\ldots,n-j+1;\\ a_{j+l,l}^{(j)}=-1, & \text{when } l=1,2,\ldots,n-j;\\ 0, & \text{otherwise.}\end{cases}$$

$$\begin{cases} h_{ii}^{(j)}=K_{j,j+i}, & i=1,2,\ldots,n-j;\\ h_{n-j+1,n-j+1}^{(j)}=s_j, & s_j=-\sum_{i=1}^{n}K_{j,i},\quad j=1,2,\ldots,n.\end{cases}$$

Let $C^{(j)}=A^{(j)}H^{(j)}A^{(j)T}$, we can find the elements of $C^{(j)}$ as

$$\begin{cases} C_{j,j}^{(j)}=K_{j,j+1}+\cdots+K_{j,n}+s_j;\\ C_{j,j+l}^{(j)}=C_{j+l,j}^{(j)}=-K_{j,j+l}, & l=1,2,\ldots,n;\\ C_{j+l,j+l}^{(j)}=K_{j,j+l}, & l=1,2,\ldots,n.\end{cases}$$

Thus $AHA^T=\sum_{j=1}^{n}C^{(j)}=-K$, the proof is completed.

References

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