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Radicals of Group G-graded Γ-rings *

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Since Nobusawa [1] proposed the conception of Γ-ring in 1964, it has been extendly studied by many researchers. In this paper we shall try to research the grading of Γ-ring.

A group G-graded Γ-ring is defined as follows:

Definition 1 *Let M be a Γ-ring and G be a group. M is called a group G-graded Γ-ring, if there exist additive sub-groups $M_g, g \in G$, of M such that*

$$M = \oplus_{g\in G} M_g$$

and

$$M_g \Gamma M_h \subseteq M_{gh}$$

for all $g, h \in G$.

Theorem 1 *Let M be a group G-graded Γ-ring and R be its right operator ring. Then R must be group a G-graded ring.*

Definition 2 *Let M be a group G-graded Γ-ring. A graded ideal Q of M is called a graded prime ideal if $A\Gamma B \subseteq Q$ implies $A \subseteq Q$ or $B \subseteq Q$ for any graded ideals A, B of M. The graded prime radical of M is defined as the intersection of all graded prime ideals of M, denoted by $N_G(M)$.*

Theorem 2 *Let M be a group G-graded Γ-ring and R be its right operator ring. Then $N_G(M) = N_G(R)^*, N_G(R) = N_G(M)^{*\prime}$.*

Definition 3 *A graded Jacobson radical, denoted by $J_G(M)$, of a graded Γ-ring M is defined by $\cap A(S)$, where S runs over all graded irreducible $M\Gamma$-modules and $J_G(M) = M$ if M has no any graded irreduible $M\Gamma$-module.*

Theorem 3 *Let M be a group G-graded Γ-ring and R be its right operator ring. Then $J_G(M) = J_G(R)^*$.*

Theorem 4 *Let M be a group G-graded Γ-ring and R be its right operator ring. Then $J_G(M)^{*\prime} = J_G(R)$.*

References

[1] N.Nobusawa, *On a generalization of ring theory,* Osaka J. Math., **1**(1964), 81–89.

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