$$G_0(c_3,c_4,\varepsilon) = c_3 + c_4 e^{-\sqrt{\frac{m}{\varepsilon}}} - a + \varphi(\varepsilon) + \frac{M}{m}\cdot\sqrt{\varepsilon},$$
$$G_1(c_3,c_4,\varepsilon) = c_3 e^{-\sqrt{\frac{m}{\varepsilon}}} + c_4 + \frac{M}{m}\cdot\sqrt{\varepsilon} - 1.$$

There exist $\varepsilon_3 > 0(\varepsilon_3 < \varepsilon_2)$ and a unique set of continuous functions $c_i(c_I(\varepsilon)$ with $0 < c_i(\varepsilon) < 2+2a$ such that for $\varepsilon \in [0,\varepsilon_3], F_i(c_1(\varepsilon), c_2(\varepsilon),\varepsilon) = G_i(c_3(\varepsilon), c_4(\varepsilon), \varepsilon) = 0, \ i = 1, 2$. Now let

$$\bar{w}(t,\varepsilon) = X_N(t,\varepsilon) + (c_1(\varepsilon)e^{-\sqrt{\frac{m}{\varepsilon}}t} + c_2(\varepsilon)e^{-\sqrt{\frac{m}{\varepsilon}}(1-t)} + \frac{M}{m}\sqrt{\varepsilon})\varepsilon^N,$$
$$\underline{w}(t,\varepsilon) = X_N(t,\varepsilon) + (c_3(\varepsilon)e^{-\sqrt{\frac{m}{\varepsilon}}t} + c_4(\varepsilon)e^{-\sqrt{\frac{m}{\varepsilon}}(1-t)} + \frac{M}{m}\sqrt{\varepsilon})\varepsilon^N$$

By [2, Lemma 5] and [3, Theroem 1], there exists an $\varepsilon_4 > 0(\varepsilon_4 < \varepsilon_3)$ such that for arbitary $\varepsilon \in (0, \varepsilon_4]$ the problem (1.1)—(1.2) has a solution satisfying $\underline{w}(t,\varepsilon) \le x(t,\varepsilon) \le \bar{w}(t,\varepsilon)$ on $t \in [0,1]$. Furthermore $x(t,\varepsilon)$ satisfies (3.1)—(3.2).

Theorem 3 *Assume that I-III, V hold, then for sufficiently small $\varepsilon > 0$, the problem (1.1)—(1.3) has a unique solution satisfying (3.1)—(3.2).*

4. An Example

Consider the prblem $\varepsilon x'' = f(t, x, W_1(\varepsilon)x', \varepsilon)$, $x(0) = x(1)$, $x'(0) = x'(1)$, where $f(t,x,y,\varepsilon)$ and $W_1(\varepsilon)$ satisfy the conditons I- III, V. In addition $f(t,x,y,\varepsilon) \equiv f(t+1,x,y,\varepsilon)$ on $t,x,y \in R^1, \varepsilon \in (0,\varepsilon_0]$.

The proof of Theorem 1 and Theorem 3 suggest $x(t,\varepsilon) \sim y_0(t) + y_2(t)\varepsilon + y_4(t)\varepsilon^2 + \cdots$ as in reference[4].

References

[1] Zhou Qinde and Miao Shumei, Northeastern Math. J., **4(3)**1988, 363-370.

[2] Wang Huaizhong and Zhou Qinde, 数学物理学报 **8**(1988), 4,389-398.

[3] L. H.Erbe, Nonlinear Analysis, Theory, Methods & Application, Vol. 6 No.11 PP. 1155-1162, 1982.

[4] 周钦德, 吉林大学自然科学学报, 1987, 第3期, 21-25.

非线性奇摄动边值问题的渐近展开

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摘　要

本文用上下解方法给出了含小参数 $\varepsilon > 0$ 的非线性奇摄动边界问题 *(1.1)—(1.2)*(周期边界问题为其特例) 解的一致有效的渐近展开式.

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Uniformly Valid Expansions of Solutions to Nonlinear Singularly Perturbed Boundary Value Problems *

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Abstract In this paper we will give uniformly valid expansions of solutions for boundary value problems (1.1)—(1.2) (containing periodic boundary condition as a special case) with a parameter $\varepsilon > 0$, by means of the upper and lower solution method.

Keyworde uniformly valid expansion, singularly perturbation, bourdary value problem, upper and lower solutions

I. Introductions

The reference[1] discussed the problem $\varepsilon x'' = f(t,x,\varepsilon)$, $L(x(0),x(1)) = 0$, $R(x(0),x(1),x'(0),x'(1)) = 0$, and gave the existence, uniqueness and an estimate of the solutions. However, these are still two problems unsolved. First, it was required that $f_x(t,x,\varepsilon)$ have a positive lower bound; secondly, the asymptotic expansions of the solution $x(t,\varepsilon)$ and its derivative $x'(t,\varepsilon)$ have not been given. In this paper, we consider the equation

$$\varepsilon x'' = f(t,x,W_1(\varepsilon)x',\varepsilon) \tag{1.1}$$

and one of the following sets of boundary conditions

$$\begin{aligned} x(0) - ax(1) &= b, \\ R(x(0),x(1),W_2(\varepsilon)x'(0),W_3(\varepsilon)x'(1)) + x'(0) - cx'(1) &= 0 \end{aligned} \tag{1.2}$$

or

$$x(0) = x(1), x'(0) = x'(1), \tag{1.3}$$

where $f(t,x,y,\varepsilon), R(x,y,u,v), W_i(\varepsilon)$ $(i = 1,2,3)$ are continuous on $0 \le t \le 1, x,y,u,v \in R, 0 \le \varepsilon \le \varepsilon_0 (\varepsilon_0 > 0)$ and a,b,c are constants with $a > 0, b > 0$.

In Section 2 we construct formal expansions of solutions to the problem (1.1)—(1.2). In Section 3 we prove the main existence and uniqueness theorems and the uniformly valid expansion. An example is given in Section 4.

Assume that

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I. $f(t,x,y,\varepsilon), R(x,y,u,v), W_i(\varepsilon)$ $(i=1,2,3)$ are suitably smooth with $w_i(0)=0$.

II. $f_x(t,x,y,\varepsilon)>0$ on $0\le t\le 1, x,y\in R, 0\le\varepsilon\le\varepsilon_0$.

III. There is a function $u(t)$ such that $f(t,u(t),0,0)=0$ on $0\le t\le 1$.

IV. $R_u(x,y,u,v)$ and $R_v(x,y,u,v)$ are bounded on R^4.

V. For every given $\varepsilon\in(0,\varepsilon_0]$, $f(t,x,y,\varepsilon)$ satisfies Nagumo's conditions[2].

Lemma 1 *Assume that conditions IV and V hold and there exist functions $w(t,\varepsilon)$ and $\underline{w}(t,\varepsilon)$ such that for $t\in[0,1],\varepsilon\in(0,\varepsilon_0]$*

$$\begin{aligned} w(t,\varepsilon) &\ge \underline{w}(t,\varepsilon),\quad \varepsilon w''(t,\varepsilon)\le f(t,w(t,\varepsilon),W_1(\varepsilon)w(t,\varepsilon),\varepsilon),\\ \varepsilon\underline{w}''(t,\varepsilon) &\ge f(t,\underline{w}(t,\varepsilon),W_1(\varepsilon)\underline{w}'(t,\varepsilon),\varepsilon)\end{aligned}$$

and

$$w(0,\varepsilon)-aw(1,\varepsilon)=b,\quad \underline{w}(0,\varepsilon)-a\underline{w}(1,\varepsilon)=b,$$

$$\begin{aligned} R(w(0,\varepsilon),w(1,\varepsilon),W_2(\varepsilon)w'(0,\varepsilon),W_3(\varepsilon)w'(1,\varepsilon))+w'(0,\varepsilon)-cw'(1,\varepsilon) &\le 0,\\ R(\underline{w}(0,\varepsilon),\underline{w}(1,\varepsilon),W_2(\varepsilon)\underline{w}'(0,\varepsilon),W_3(\varepsilon)\underline{w}'(1,\varepsilon))+\underline{w}'(0,\varepsilon)-c\underline{w}'(1,\varepsilon) &\ge 0.\end{aligned}$$

Then there exists an $\varepsilon_2\in(0,\varepsilon_0]$ such that for every $\varepsilon\in(0,\varepsilon_2]$, problem (1.1)—(1.2) has a solution $x(t,\varepsilon)$ satisfying $\underline{w}(t,\varepsilon)\le x(t,\varepsilon)\le w(t,\varepsilon)$ on $t\in[0,1]$.

For the proof, cf. [3, Theroem 3.1].

2. Constructing Formal Expansions

It is easy to see that the problem $\varepsilon x''=\varepsilon x'+x+t$, $x(0)=x(1)$, $x'(0)=x'(1)$ has a unique solution $x(t,\varepsilon)$ satisfying $x(t,\varepsilon)\sim-\frac{1}{2}e^{-(\frac{t}{\sqrt{\varepsilon}})}+\frac{1}{2}e^{-\frac{1-t}{\sqrt{\varepsilon}}}-t+\varepsilon$ as $\varepsilon\to 0^+$ This suggests that the solution of (1.1)—(1.2) have the form

$$x(t,\varepsilon)=y(t,\varepsilon)+u(\tau,\mu)+v(s,\mu), \tag{2.1}$$

where $y(t,\varepsilon)$ is the regular part, $u(\tau,\mu)$ and $v(s,\mu)$ are the boundary layer functions, $\tau=\frac{t}{\mu}, s=\frac{1-t}{\mu}, \mu=\sqrt{\varepsilon}$, and $y(t,\varepsilon)=y_0(t)+y_2(t)\varepsilon+y_4(t)\varepsilon^2+\cdots, u(\tau,\mu)=u_0(\tau)+u_1(\tau)\mu+\cdots, v(s,\mu)=v_0(s)+v_1(s)\mu+v_2(s)\mu^2+\cdots$.

Substituting equality (2.1) into (1.1), we have

$$\begin{aligned}&\varepsilon y''(t,\varepsilon)+u''(\tau,\mu)+v''(s,\mu)=f(t,y(t,\varepsilon),W_1(\varepsilon)y'(t,\varepsilon),\varepsilon)\\ &+[f(t,y(t,\varepsilon))+u(\tau,\mu),W_1(\varepsilon)(y'(t,\varepsilon)+\frac{1}{\mu}u'(\tau,\mu),\varepsilon)\\ &-f(t,y(t,\varepsilon),W_1(\varepsilon)y'(t,\varepsilon))]\Big|_{\varepsilon=\mu^2}^{t=\tau\mu}+[f(t,y(t,\varepsilon)+u(\tau,\mu)+v(s,\mu)),\\ &W_1(\varepsilon)(y'(t,\varepsilon)+\frac{1}{\mu}u'(\tau,\mu)-\frac{1}{\mu}v'(s,\mu),\varepsilon)-f(t,y(t,\varepsilon)+u(\tau,\mu),\\ &W_1(\varepsilon)(y'(t,\varepsilon)+\frac{1}{\mu}u'(\tau,\mu),\varepsilon)]\Bigg|_{\substack{t=1-\mu s\\ \tau=\frac{1}{\mu}-s\\ \varepsilon=\mu^2}}\end{aligned}$$

Expanding formally we obtain the equations:

$$f(t, y_0(t), 0, 0) = 0, \tag{2.2}$$

$$\begin{aligned} y''_{i-2}(t) &= f_x(t, y_0(t), 0, 0)y_i(t) \\ &\quad + P_i(t, y_0(t), y'_0(t), \cdots, y_{i-2}(t), y'_{i-2}(t)) \quad (i = 2, 4, 6, \cdots), \end{aligned} \qquad (2.2)_i$$

$$u''_0(\tau) = \int_0^1 f_x(0, y_0(0) + \theta u_0(\tau), 0, 0)d\theta u_0(\tau), \qquad (2.3)_0$$

$$\begin{aligned} u''_k(\tau) &= f_x(0, y_0(0) + u_0(\tau), 0, 0)u_k(\tau) \\ &\quad + Q_k(\tau, u_1(\tau), u'_1(\tau), \cdots, u_{k-1}(\tau), u'_{k-1}(\tau)), \end{aligned} \qquad (2.3)_k$$

$$v''_0(s) = \int_0^1 f_x(1, y_0(1) + \theta v_0(s), 0, 0)d\theta v_0(s), \qquad (2.4)_0$$

$$\begin{aligned} v''_k(s) &= f_x(1, y_0(1) + v_0(s), 0, 0)v_k(s) \\ &\quad + R_k(s, v_0(s), v'_0(s), \cdots, v_{k-1}(s), v'_{k-1}(s)), \end{aligned} \qquad (2.4)_k$$

where P_i is a known function of $t, y_0(t), y'_0(t), \cdots, y_{i-2}(t), y'_{i-2}(t)$; Q_k is a known fuction of $\tau, u_0(\tau), u'_0(\tau), \cdots, u_{i-1}(\tau), u'_{i-1}(\tau)$, in particular, when $u_1(\tau)$ is bounded, each Q_k can be written as a polynomial of $u_0(\tau), u'_0(\tau), \cdots, u_{k-1}(\tau), u'_{k-1}(\tau)$ without constant terms, all of whose coefficients can be respresented by finite sums of the form $\sum a_i(\tau)\tau^i$ with $a_i(\tau)$ being bounded functions of τ; and R_k a known function of $s, v_0(s), v'_0(s), \cdots, v_{k-1}(s), v'_{k-1}(s)$ in the similar form as Q_k.

Substituting (2.1) into the boundary condition (1.2) we get the conditions of determining solutions:

$$u_i(0) - av_i(0) = b_i, \quad u'_i(0) + cv'_i(0) = d_i, \qquad (2.5)_i$$

$$u_i(+\infty) = 0, \qquad v_i(+\infty) = 0, \qquad (2.6)_i$$

where b_i is a known function of $y_i(0)$ and $y_i(1)$, and d_i is a known function of $y_k^{(j)}(0)$, $y_k^{(j)}(1)$, $u_k^{(j)}(0)$ and $v_k^{(j)}(0)$ with $0 \le k \le i-1, i = 1, 2, 3\cdots, j = 0, 1$ and $d_0 = 0$.

In view of II, III and $(2.2)_i$, we obtain successively the function $y_i(t)$ on $0 \le t \le 1$:

$$y_i(t) = [f_x(t, y_0(t), 0, 0)]^{-1}\left[y''_{i-2} - P_i(t, y_0(t), \cdots, y_{i-2}(t))\right], \quad (i = 2, 4, \cdots).$$

Theorem 1 *Assume that condition I—III are satisfied, then we can determine the functions $u_k(t), v_k(t)$ on $t \ge 0$ successively from $(2.3)_k$—$(2.6)_k$. Furthermore, there exists a positive number sequence $\{M_i\}_{i=0}^{\infty}$ independent of μ, such that*

$$|u_i(t)| + |u'_i(t)| \le M_i e^{-\sigma t} \text{ on } t \ge 0, \qquad (2.7)_i$$

$$|v_i(t)| + |v_i'(t)| \le M_i e^{-\sigma t} \text{ on } t \ge 0, \tag{2.8$_i$}$$

where σ is a positive constant.

Proof In fact, we can prove

$$|u_i(t)| + |u_i'(t)| \le M_i e^{-\sigma t(1+\frac{1}{2^i})} \text{ on } t \ge 0, \tag{2.7$'_i$}$$

$$|v_i(t)| + |v_i'(t)| \le M_i e^{-\sigma t(1+\frac{1}{2^i})} \text{ on } t \ge 0. \tag{2.8$'_i$}$$

Here we will only prove that the problem $(2.3)_0$—$(2.6)_0$ has a unique solution (u_0, v_0) satisfying $(2.7)'_i$—$(2.8)'_i$, since similar method and induction can prove the remaining parts. To this end, consider the problem

$$x'' = \int_0^1 f_x(0, y_0(0) + \theta x, 0, 0) d\theta x, x(0) \text{ fixed }, x(+\infty) = 0. \tag{2.9}$$

Let $m = \min\limits_{|y| \le |x(0)|} f_x(0, y_0(0) + y, 0, 0)$, in view of [2, Lemma 1] we obtain that the problem (2.9) has a unique solution $x(t)$ such that $|x(t)| \le |x(0)| e^{-\sqrt{m}t}$ on $t \in [0, +\infty)$, $x(+\infty) = 0$. And we can obtain that the unique solution $x(t)$ of (2.9) has the following properties: $x'(0)$ is a strictly decreasing function with respect to $x(0)$ and

$$|x(t)| \le |x(0)| \cdot e^{-\sqrt{m}t},$$

$$|x'(t)| \le \max_{|y| \le |x(0)|} |f_x(0, y_0(0) + y, 0, 0)| \cdot \frac{|x(0)|}{\sqrt{m}} \cdot e^{-\sqrt{m}t}, \tag{2.10}$$

$$x'(0) \cdot x(0) \le 0. \tag{2.11}$$

Similary we can show that the problem

$$y'' = \int_0^1 f_x(0, y_0(0) + \theta y, 0, 0) d\theta y, y(0) \text{ fixed }, y(+\infty) = 0$$

has a unique solution such that

$$|y(t)| \le |y(0)| \cdot e^{-\sqrt{m_2}t},$$

$$|y'(t)| \le \max_{|y| \le |y(0)|} |f_x(1, y_0(1) + y, 0, 0)| \cdot \frac{|y(0)|}{\sqrt{m_2}} \cdot e^{-\sqrt{m_2}t}, \tag{2.12}$$

where $m_2 = \min\limits_{|y| \le |y(0)|} f_x(1, y_0(1) + y, 0, 0)$. And $y'(0)$ is a strictly decreasing continuous function with respect to $y(0)$, and

$$y(0) \cdot y'(0) \le 0. \tag{2.13}$$

Now, let $x(0) = ay(0) + b, I(y(0)) = x'(0) + cy'(0)$. Then $I(y(0))$ is a strictly decreasing continuous function, and if $y(0) > 0$ with $x(0) > 0$ then $I(y(0)) \le 0$, if $y(0) < 0$ with $x(0) < 0$ then $I(y(0)) \ge 0$. Thus there exists a unique value $\bar{y}(0)$ of $y(0)$ such that

$I(\bar{y}(0)) = 0$. At this time, denote $x(t)$ and $y(t)$ by $u_0(t)$ and $v_0(t)$ respectively, then $(u_0(t), v_0(t))$ is the unique solution of $(2.3)_0$—$(2.6)_0$. Furthermore, let

$$\begin{aligned}(2\sigma)^2 &= \min\left[\min_{|y|\le|\bar{y}(0)|} f_x(1, y_0(1)+y, 0, 0), \min_{|y|\le|a+\bar{y}(0)b_0|} f_x(0, y_0(0)+y, 0, 0)\right],\\ M_0 &= |\bar{y}(0)| + |a\bar{y}(0)+b_0| + \max_{|y|\le|y(0)|}|f_x(1, y_0(0)+y, 0, 0)|\cdot\frac{|\bar{y}(0)|}{\sigma}\\ &\quad + \max_{|y|\le|a\bar{y}(0)+b_0|}|f_x(0, y_0(0)+y, 0, 0)|\cdot\frac{|a\bar{y}(0)+b_0|}{\sigma}.\end{aligned}$$

Then in view of (2.10) and (2.12), $u_0(t)$ and $v_0(t)$ satisfies $(2.7)'_0$ and $(2.8)'_0$ respectively.

3. Main results

Theorem 2 *Assume that conditions I–V are satisfied, then for sufficiently small $\varepsilon > 0$, the problem (1.1)—(1.2) has a solution $x(t,\varepsilon)$ satisfying*

$$|x(t,\varepsilon) - X_N(t,\varepsilon)| \le K\varepsilon^N(\varepsilon^{\frac{1}{2}} + e^{-\sqrt{\frac{m}{\varepsilon}}t} + e^{-\sqrt{\frac{m}{\varepsilon}}(1-t)}), \tag{3.1}$$

$$|x'(t,\varepsilon) - X'_N(t,\varepsilon)| \le K\varepsilon^{N-\frac{1}{2}}, \tag{3.2}$$

where $X_N(t,\varepsilon) = \sum_{i=0}^{2N}[y_i(t) + u_i(\frac{t}{\sqrt{\varepsilon}}) + v_i(\frac{1-t}{\sqrt{\varepsilon}})]\varepsilon^{\frac{i}{2}}$, N is a nonnegative integer, K is a positive constant independent of ε, $y_i(t), u_i(t)$ and $v_i(t)$ are functions determined by Theorem 1, and

$$\begin{aligned} m &= \min\{f_x(t,x,y,\varepsilon) | t\in[0,1], |x - y_0(t)|\\ &\qquad \le |v_0(0)| + |u_0(0)| + 1, |y| \le 1, \varepsilon\in[0,\varepsilon_0]\},\\ y_1(t) &\equiv y_3(t) \equiv \cdots \equiv y_{2N-1}(t) \equiv 0.\end{aligned}$$

Proof The proof is adopted from the proof of [1, theorem].

Let $\varphi(\varepsilon) = [X_N(0,\varepsilon) - aX_N(1,\varepsilon) - b]\cdot\varepsilon^{-N}$ Then it is easy to see that $\varphi(\varepsilon) = O(\sqrt{\varepsilon})$. It is also clear that there exists $M_1 > 0$ such that $|f(t, X_N(t,\varepsilon), W_1(\varepsilon)X'(t,\varepsilon)\varepsilon) - \varepsilon X''_N(t,\varepsilon)| \le M_1\varepsilon^{N+\frac{1}{2}}$ on $t\in[0,1], \varepsilon\in[0,\varepsilon_0]$. We can suppose $W_1(\varepsilon) \sim W_{11} + W_{12}\varepsilon^2 + \cdots$. Let

$$\begin{aligned} M_2 &= (|W_{11}| + 1)(4a + 4),\\ M_3 &= \max\{f_x(t,x,y,\varepsilon) | t\in[0,1], |x - y_0(t)|\\ &\qquad \le |u_0(0)| + v_0(0) + 1, |y| \le 1, \varepsilon\in[0,\varepsilon_0]\}\\ M &= M_1 + M_2M_3.\end{aligned}$$

And let

$$\begin{aligned} F_0(c_1,c_2,\varepsilon) &= c_1 + c_2e^{-\sqrt{\frac{m}{\varepsilon}}} - a + \varphi(\varepsilon) + \frac{M}{m}\cdot\sqrt{\varepsilon},\\ F_1(c_1,c_2,\varepsilon) &= c_1e^{-\sqrt{\frac{m}{\varepsilon}}} + c_2 - 1 + \frac{M}{m}\cdot\sqrt{\varepsilon},\end{aligned}$$

$$G_0(c_3,c_4,\varepsilon) = c_3+c_4e^{-\sqrt{\frac{m}{\varepsilon}}}-a+\varphi(\varepsilon)+\frac{M}{m}\cdot\sqrt{\varepsilon},$$
$$G_1(c_3,c_4,\varepsilon) = c_3e^{-\sqrt{\frac{m}{\varepsilon}}}+c_4+\frac{M}{m}\cdot\sqrt{\varepsilon}-1.$$

There exist $\varepsilon_3>0(\varepsilon_3<\varepsilon_2)$ and a unique set of continuous functions $c_i(c_I(\varepsilon)$ with $0<c_i(\varepsilon)<2+2a$ such that for $\varepsilon\in[0,\varepsilon_3], F_i(c_1(\varepsilon),c_2(\varepsilon),\varepsilon)=G_i(c_3(\varepsilon),c_4(\varepsilon),\varepsilon)=0,\ \ i=1,2.$ Now let

$$w(t,\varepsilon) = X_N(t,\varepsilon)+(c_1(\varepsilon)e^{-\sqrt{\frac{m}{\varepsilon}}t}+c_2(\varepsilon)e^{-\sqrt{\frac{m}{\varepsilon}}(1-t)}+\frac{M}{m}\sqrt{\varepsilon})\varepsilon^N,$$
$$\underline{w}(t,\varepsilon) = X_N(t,\varepsilon)+(c_3(\varepsilon)e^{-\sqrt{\frac{m}{\varepsilon}}t}+c_4(\varepsilon)e^{-\sqrt{\frac{m}{\varepsilon}}(1-t)}+\frac{M}{m}\sqrt{\varepsilon})\varepsilon^N$$

By [2, Lemma 5] and [3, Theroem 1], there exists an $\varepsilon_4>0(\varepsilon_4<\varepsilon_3)$ such that for arbitary $\varepsilon\in(0,\varepsilon_4]$ the problem (1.1)—(1.2) has a solution satisfying $\underline{w}(t,\varepsilon)\le x(t,\varepsilon)\le w(t,\varepsilon)$ on $t\in[0,1]$. Furthermore $x(t,\varepsilon)$ satisfies (3.1)—(3.2).

Theorem 3 *Assume that I-III, V hold, then for sufficiently small $\varepsilon>0$, the problem (1.1)—(1.3) has a unique solution satisfying (3.1)—(3.2).*

4. An Example

Consider the prblem $\varepsilon x''=f(t,x,W_1(\varepsilon)x',\varepsilon)$, $x(0)=x(1)$, $x'(0)=x'(1)$, where $f(t,x,y,\varepsilon)$ and $W_1(\varepsilon)$ satisfy the conditons I- III, V. In addition $f(t,x,y,\varepsilon)\equiv f(t+1,x,y,\varepsilon)$ on $t,x,y\in R^1,\varepsilon\in(0,\varepsilon_0]$.

The proof of Theorem 1 and Theorem 3 suggest $x(t,\varepsilon)\sim y_0(t)+y_2(t)\varepsilon+y_4(t)\varepsilon^2+\cdots$ as in reference[4].

References

[1] Zhou Qinde and Miao Shumei, Northeastern Math. J., **4(3)**1988, 363–370.

[2] Wang Huaizhong and Zhou Qinde, 数学物理学报 **8**(1988), 4,389–398.

[3] L. H.Erbe, Nonlinear Analysis, Theory, Methods & Application, Vol. 6 No.11 PP. 1155–1162, 1982.

[4] 周钦德, 吉林大学自然科学学报, 1987, 第3期, 21–25.

非线性奇摄动边值问题的渐近展开

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摘　要

本文用上下解方法给出了含小参数 $\varepsilon>0$ 的非线性奇摄动边界问题(1.1)—(1.2)(周期边界问题为其特例)解的一致有效的渐近展开式.