then by (6)

$$|\Phi_2(z) - \Phi_2^+(t)| = |\phi(t_z) - \phi(t)| \le C_3|t-z|^\nu, \quad t \in \Omega.$$

when z is close to another point t_0 on Ω from the inner of D^+, we have

$$|\Phi_2^+(t_0) - \Phi_2^+(t)| \le C_3|t-t_0|^\nu, \quad t_0, t \in \Omega.$$

For $z \in D^-, \Phi_2^-(t) = 0$, correspondly there is

$$|\Phi_2(z) - \Phi_2^-(t)| = 0, \quad t \in \Omega.$$

From the above, we have got (14).

Notice that if Ω is smooth close surface, then by Plemelj formulla there is $\Phi(t) = \frac{1}{2}[\Phi^+(t)+\Phi^-(t)]$. So by theorem 2 we have the result that if $\phi(\xi)$ satisfies Hölder condition on Ω, then $\Phi(t)$ also satisfies Hölder condition. But if Ω is piecewise smooth closed surface (that is Ω has angular points and sharp points), the result is not right.

Since

$$\Phi(t) = \int_\Omega \phi(\xi)K(\xi,t) = \frac{1}{2}[\Phi^+(t) + \Phi^-(t)] + (1 - \frac{\beta(t)}{s})\phi(t),$$

where $\beta(t)$ is not a continuous function.

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关于多复变数 Cauchy 型积分的边界性质

陈 叔 瑾
(厦门大学研究生院,厦门 361005)

摘 要

本文用新的方法研究 B-M 型积分的边界性质,所得结果推进了文[1]的结果,并指出文[4]证明有错误.

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The Boundary Properties of Cauchy Type Integral in Several Complex Variables *

Chen Shujin
(Xiamen University, Xiamen, 361005)

Abstract We study boundary properties of B-M type intgral using a new method. We generalize the result of [1], and point out an error in [4].

Key words Cauchy type integral, boundary properties

1. Introduction

Bochner-Martinelli (abbr. B-M below) integral representation has a very important place in functions of several complex variables, for this representation is applicable to general bounded domains with piecewise smooth boundaries in C^n space. Naturaly, we hope to consider the corresponding integral as in functions of one complex variable. [1] and [2] discuss the boundary properties of a B-M type integral on a bounded domain with c^2 smooth boundary. Using the method parallel to [1] and [3], paper [4] discussed the case when boundary has angular- points, but there is an error in the proof. This paper will study boundary properties of B-M type integral in 2 and 3 using a new method. This method not only simplities the proof of paper [1], but also develops the result of paper [1] to bounded domains with piecewise smooth boundary (even to non-closed piecewise smooth surface).

2. Plemelj Formula

Let $\beta(t)$ represent the measure of the solid angle when we look at the surface Ω from point t (that is it is the area of the piece of the surface(positive side) cut by Ω while we draw an unit sphere and use point t as the center). Then corresponding to the case for functions of one complex variable we can get in C^n space:

Theorem 1 *If D is a domain with piecewise smooth boundaries Ω in space C^n, function $\phi(\xi)$ satisfies Hölder condition on Ω, then for any $t \in \Omega$, we have Plemelj formula:*

$$\Phi^+(t) = (1 - \frac{\beta(t)}{s})\phi(t) + \int_\Omega \phi(\xi)K(\xi,t), \tag{1}$$

$$\Phi^-(t) = -\frac{\beta(t)}{s}\phi(t) + \int_\Omega \phi(\xi)K(\xi,t), \tag{2}$$

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Where s is the area of a unit sphere in C^n, *that is* $s = 2\pi^n/(n-1)!, 0 \le \beta(t) \le S. K(\xi, t)$ *is the kernel of B-M integral.*

In order to get theorem 1, we need the following tow lemmas.

Lemma 1 *If D is a domain with piecewise smooth boundaries* Ω *in* C^n, *then for any* $t \in \Omega$, *we have:*

$$\int_{\Omega} K(\xi, t) = \lim_{\epsilon \to 0} \int_{\Omega - \omega_\epsilon} K(\xi, t) = \beta(t)/s, \tag{3}$$

where $\omega_\epsilon = \Omega \cap B_\epsilon(t), B_\epsilon(t) = \{\xi \in C^n : |\xi - t| < \epsilon\}$.

Proof Consider the domain D_0 with boundaries $\Omega - \omega_\epsilon$ and $\partial B_\epsilon(t) \cap D$. Then we have $dK(\xi, t) = 0$ on D_0, thus applying Stokes theorem we have:

$$\int_{\Omega - \omega_\epsilon} K(\xi, t) = \int_{\partial B_\epsilon(t) \cap D} K(\xi, t).$$

set $\xi_j = \varsigma_j + i\eta_j, t_j = x_j + iy_j, j = 1, 2, \cdots, n$. then $d\bar{\xi}_j \wedge d\xi_j = 2i d\varsigma_j \wedge d\eta_j$ (omit the sign $\wedge$ below). So on $\partial B_\epsilon(t) \cap D$:

$$\begin{aligned} K(\xi, t) &= \frac{(n-1)!}{(2\pi i)^n} \frac{(2i)^{n-1}}{\epsilon^{2n-1}} \sum_{j=1}^{n} \Big(\frac{\varsigma - x_j}{|\xi - t|} d\varsigma_1 d\eta_1 \cdots [d\varsigma_j] d\eta_j \cdots d\varsigma_n d\eta_n \\ &\quad - \frac{\eta_j - y_j}{|\xi - t|} d\varsigma_1 d\eta_1 \cdots d\varsigma_j [d\eta_j] \cdots d\varsigma_n d\eta_n \Big) = \frac{(n-1)!}{(2\pi i)^n} \frac{i(2i)^{n-1}}{\epsilon^{2n-1}} d\sigma \\ &= \frac{1}{s} \cdot \frac{d\sigma}{\epsilon^{2n-1}}, \end{aligned}$$

where $d\sigma$ is the element of area of sphere. Then

$$\int_{\Omega} K(\xi, t) = \lim_{\epsilon \to 0} \int_{\partial B_\epsilon(t) \cap D} K(\xi, t) = \lim_{\epsilon \to 0} \frac{1}{s} \frac{1}{\epsilon^{2n-1}} \int_{\partial B_\epsilon(t) \cap D} d\sigma.$$

Notice the edge of $\partial B_\epsilon(t) \cap D$ is piecewise smooth for Ω is a piecewise smooth surface. Obviously for every $\epsilon > 0, \partial B_\epsilon(t) \cap D$ is measurable. So using spherical coordinates we easily get

$$\int_{\partial B_\epsilon(t) \cap D} d\sigma = O(\epsilon^{2n-1}) \text{ and } \lim_{\epsilon \to 0} \Big(\int_{\partial B_\epsilon(t) \cap D} d\sigma / \epsilon^{2n-1} \Big) = \beta(t),$$

where $0 \le \beta(t) \le s$.

Lemma 2 *If* Ω *is a piecewise smoothly closed surface,* $\phi(\xi)$ *satisfies Hölder condition with exponent* ν *on* Ω. *Then*

$$\Big| \int_{\omega_\delta} [\phi(\xi) - \phi(t_z)] K(\xi, z) \Big| \le O(\delta^\nu), \quad z \in \bar{D}, \tag{4}$$

where $t_z \in \omega_\delta$, *and such that* $|t_z - z| = \inf_{\xi \in \omega_\delta} |\xi - z|$ *(cf.[5]).*

Proof First consider $z \notin \Omega$. Since $\phi(\xi)$ satisfies Hölder condition with exponent ν on Ω. We have

$$|\phi(\xi) - \phi(t_z)| \le C_1|\xi - t_z|^{\nu}.$$

Also since

$$|\xi - t_z| \le |\xi - z| + |z - t_z| \le 2|\xi - z|, \tag{5}$$

we have

$$|\phi(\xi) - \phi(t_z)| \le C_2|\xi - z|^{\nu}, \tag{6}$$

C_1, C_2 are positive constants. Thus, we get

$$\begin{aligned} |\int_{\omega_\delta}[\phi(\xi) - \phi(t_z)]K(\xi, z)| &= |\int_{\omega_z}[\phi(\xi) - \phi(t_z)]\sum_{j=1}^{n}\frac{\bar{\xi}_j - \bar{z}_j}{|\xi - z|^{2n}} \cdot \lambda_j d\sigma| \\ &\le C_3\int_{\omega_\delta}\frac{d\sigma}{|\xi - t_z|^{2n-\nu-1}} \quad (C_3 \text{ is positive constant}). \end{aligned}$$

Now first supposing that ω_δ is smooth, we can represent ω_δ using real coordinates in a neighborhood of point t:

$$\xi_1 - x_1 = \psi(\varsigma_2 - x_2, \cdots, \varsigma_n - x_n, \eta_1 - y_1, \cdots, \eta_n - y_n),$$

where $(x_1, \cdots, x_n, y_1, \cdots, y_n)$ is the real coodinates of point t, ψ is smooth function. Take the positive side of ω_δ, represent θ as the angle limited by the normal direction and positive direction of ς_1 axis, then $0 \le \theta \le \theta_0 < \pi/2$, and

$$d\varsigma_2 \cdots d\varsigma_n d\eta_1 \cdots d\eta_n = \cos\theta d\sigma = d\sigma \Bigg/ \sqrt{1 + \sum_{j=2}^{n}(\frac{\partial\phi}{\partial\varsigma_j})^2 + \sum_{j=1}^{n}(\frac{\partial\phi}{\partial\eta_j})^2}.$$

Denote

$$M = \sup_{\omega_\delta}\sqrt{1 + \sum_{j=2}^{n}(\frac{\partial\phi}{\partial\varsigma_j})^2 + \sum_{j=1}^{n}(\frac{\partial\phi}{\partial\eta_j})^2},$$

then

$$d\sigma \le M d\varsigma_2 \cdots d\varsigma_n d\eta_1 \cdots d\eta_n.$$

Now use spherical coordinates:

$$\begin{aligned} &\varsigma - \bar{x}_1 = \varsigma - \bar{x}_1, \quad \varsigma_2 - \bar{x}_2 = r\cos\theta_1, \quad \varsigma_3 - \bar{x}_3 = r\sin_1\theta, \cos\theta_2, \\ &\quad \cdots, \\ &\eta_{n-1} - \bar{y}_{n-1} = r\sin\theta_1\cdots\sin\theta_{2n-3}\sin\theta_{2n-2}, \eta_n - \bar{y}_n = r\sin\theta_1\cdots\sin\theta_{2n-3}\sin\theta_{2n-2}, \end{aligned}$$

where $(\bar{x}_1, \cdots, \bar{x}_n, \bar{y}_1, \cdots, \bar{y}_n)$ is the real coordinates of t_z, then

$$\begin{aligned}\int_{\omega_\delta} \frac{d\sigma}{|\xi - t_z|^{2n-\nu-1}} &\leq M \int_0^\pi \cdots \int_0^\pi \int_0^{2\pi} \sin^{2n-3}\theta_1 \cdots \sin\theta_{2n-3} d\theta_1 \cdots d\theta_{2n-2} \\ &\quad \cdot \int_0^{2\delta} r^{\nu-1} dr \leq O(\delta^\nu).\end{aligned}$$

So (4) is right when ω_δ is smooth.

Moreover, if ω_δ has angular points (it means that $0 < \beta(t) < s, \beta(t) \neq s/2$) and sharp points (it means that $\beta(t) = 0$ or s), we divide ω_δ into several pieces of smooth surface without angular points (sharp points), then apply the above result on each piece of smooth surface. Just the same we get (4) since the integral on ω_δ is nothing but the sum of integrals on these pieces of smooth surface.

Finally, for $z \in \Omega$ we notice (5) and (6). Just the same as the above we know that the integral of the left of (4) is still a convergent generalized integal, so (4) is also right.

Now the following prove theorem 1. In face, we write (in convenience, denote $D = D^+, C^m \setminus D = D^-$):

$$\Phi(z) = \int_\Omega [\phi(\xi) - \phi(t_z)] K(\xi, z) + \phi(t_z) \int_\Omega K(\xi, z), \quad z \in D^\pm, \tag{7}$$

where $t_z \in \omega_\delta$, and such that $|t_z - z| = \inf_{\xi \in \omega_\delta} |\xi - z|$. According to B-M integral formula of holomorphic functions, we have $\int_\Omega K(\xi, z) = 1$ for $z \in D^+, \int_\Omega K(\xi, z) = 0$ for $z \in D^-$ Then

$$\lim_{\substack{z \to t \\ z \in D^+}} \phi(t_z) \cdot \int_\Omega K(\xi, z) = \phi(t), \tag{8}$$

$$\lim_{\substack{z \to t \\ z \in D^-}} \phi(t_z) \cdot \int_\Omega K(\xi, z) = 0. \tag{9}$$

On the other hand,

$$\begin{aligned}&|\int_\Omega [\phi(\xi) - \phi(t_z)] K(\xi, z) - \int_\Omega [\phi(\xi) - \phi(t)] K(\xi, t)| \\ &\leq |\int_{\Omega - \omega_\delta} [\phi(\xi) - \phi(t_z)] K(\xi, z) - \int_{\Omega - \omega_\delta} [\phi(\xi) - \phi(t)] K(\xi, t)| \\ &\quad + |\int_{\omega_\delta} [\phi(\xi) - \phi(t_z)] K(\xi, z)| + |\int_{\omega_\delta} [\phi(\xi) - \phi(t)] K(\xi, t)| \\ &= I_1 + I_2 + I_3.\end{aligned}$$

By lemma 2, we get $I_2 = O(\delta^\nu), I_3 = O(\delta^\nu)$. Thus for any $\epsilon > 0$, we only have to take enough small $\delta > 0$, then there are $I_2 < \epsilon/3$ and $I_3 < \epsilon/3$. For such a δ, we proceed to estimate I_1. Since

$$I_1 = |\int_{\Omega - \omega_\delta} [\phi(\xi) - \phi(t)] K(\xi, z) + \int_{\Omega - \omega_\delta} [\phi(t) - \phi(t_z)] K(\xi, z)$$

$$
\begin{aligned}
& -\int_{\Omega-\omega_\delta}[\phi(\xi)-\phi(t)]K(\xi,t)| \le |\int_{\Omega-\omega_\delta}[\phi(\xi)-\phi(t)][K(\xi,z)-K(\xi,t)]| \\
& +|\int_{\Omega-\omega_\delta}[\phi(\xi)-\phi(t_z)]K(\xi,z)| \le |\int_{\Omega-\omega_\delta}[\phi(\xi)-\phi(t)][K(\xi,z)-K(\xi,t_z)]| \\
& +|\int_{\Omega-\omega_\delta}[\phi(\xi)-\phi(t)][K(\xi,t_z)-K(\xi,t)]| + |\phi(t)-\phi(t_z)||\int_{\Omega-\omega_\delta}K(\xi,z)| \\
= \;& I_1' + I_1'' + I_1'''.
\end{aligned}
$$

First we estimate I_1'. We consider

$$
\begin{aligned}
& |K(\xi,z)-K(\xi,t_z)| = |\frac{(n-1)!}{2\pi^n i}\sum_{j=1}^{n}\lambda_j(\frac{\bar{\xi}_j-\bar{z}_j}{|\xi-z|^{2n}} - \frac{\bar{\xi}_j-\bar{t}_{z_j}}{|\xi-t_z|^{2n}})d\sigma| \\
\le\; & \frac{(n-1)!}{2\pi^n}[|\sum_{j=1}^{n}(\frac{1}{|\xi-z|^{2n}} - \frac{1}{|\xi-t_z|^{2n}})\lambda_j(\bar{\xi}_j-\bar{z}_j)d\sigma| + |\sum_{j=1}^{n}\frac{\bar{t}_{z_j}-\bar{z}_j}{|\xi-t_z|^{2n}}\lambda_j d\sigma|] \\
\le\; & \frac{(n-1)!}{2\pi^n}(\frac{|\xi-t_z|^{2n}-|\xi-z|^{2n}}{|\xi-z|^{2n-1}|\xi-t_z|^{2n}} + \frac{|t_z-z|}{|\xi-t_z|^{2n}})d\sigma \\
=\; & \frac{(n-1)!}{2\pi^n}[\frac{(|\xi-t_z|-|\xi-z|)(|\xi-t_z|^{2n-1}+|\xi-t_z|^{2n-2}|\xi-z|+\cdots+|\xi-z|^{2n-1})}{|\xi-z|^{2n-1}|\xi-t_z|^{2n}} \\
& +\frac{|t_z-z|}{|\xi-t_z|^{2n}}]d\sigma \\
\le\; & \frac{(n-1)!}{2\pi^n}\frac{|t_z-z|}{|\xi-t_z|^{2n}}(\frac{|\xi-t_z|^{2n-1}+|\xi-t_z|^{2n-2}|\xi-z|+\cdots+|\xi-z|^{2n-2}}{|\xi-z|^{2n-1}}+1)d\sigma \\
\le\; & \frac{(n-1)!}{2\pi^n}\frac{|t_z-z|}{|\xi-t_z|^{2n}}[2+\sum_{j=1}^{2n-1}(\frac{|\xi-z|+|z-t_z|}{|\xi-z|})^j]d\sigma \le C_4\frac{|t_z-z|}{|\xi-t_z|^{2n}}d\sigma,
\end{aligned}
$$

where C_4 is positive constant. As z is sufficiently close to t, for example $|z-t|<\delta/4, |z-t_z| = \inf|z-\xi| < |z-t| < \delta/4$, for $\xi\in\Omega-\omega_\delta$, we have

$$
|\xi-t_z| \ge |\xi-t|-|t-t_z| \ge |\xi-t|-(|t-z|+|z-t_2|) \ge \frac{\delta}{2}
$$

and $|\xi-t| = |\xi-t_z+t_z-t| \le |\xi-t_z|+|t_z-t| \le 2|\xi-t_z|$. then

$$
\begin{aligned}
I_1' \;&\le\; C_5\int_{\Omega-\omega_\delta}\frac{|\xi-t|^\nu|t_z-z|}{|\xi-t_z|^{2n}}d\sigma \le 2C_5|t_z-z|\int_{\Omega-\omega_\delta}\frac{d\sigma}{|\xi-t_z|^{2n-\nu}} \\
&\le\; 2C_5\frac{|t_z-z|}{(\frac{\delta}{2})^{2n-\nu}}\int_{\Omega-\omega_\delta}d\sigma \le C_6|t_z-z| \le C_6|t-z|,
\end{aligned}
$$

where C_5 and C_6 are positive constants. Thus when z is sufficiently close to t, there is $I_1' < \delta/9$.

In order to estimate I_1'', only notice that when $\xi\in\Omega-\omega_\delta$, there is $|\xi-t_z| < |\xi-t| + |t-t_z|$, also when z is sufficiently close to t, t_z can be sufficiently close to t, so there is

$|t-t_z|<|\xi-t|$. As the above inference, we have

$$|K(\xi,t_z)-K(\xi,t)|\le\frac{(n-1)!}{2\pi^n}\frac{|t_z-t|}{|\xi-t_z|^{2n}}[2+\sum_{j=1}^{2n-1}(\frac{|\xi-t_z|}{|\xi-t|})^j]d\sigma$$
$$\le\frac{(n-1)!}{2\pi^n}\frac{|t_z-t|}{|\xi-t_z|^{2n}}[2+\sum_{j=1}^{2n-1}(1+\frac{|t-t_z|}{|\xi-t|})^j]d\sigma\le C_7\frac{|t_z-t|}{|\xi-t_z|^{2n}}d\sigma,$$

where C_7 is positive constant. Thus,

$$I_1''\le C_8\int_{\Omega-\omega_\delta}\frac{|\xi-t|^\nu|t_z-t|}{|\xi-t_z|^{2n}}d\sigma\le C_8|t_z-t|\int_{\Omega-\omega_\delta}\frac{d\sigma}{|xi-t_z|^{2n-1}},$$

where C_8 is positive constant. Since $|t_z-t|\le|t_z-z|+|z-t|\le 2|z-t|$, we have $I_1''<\epsilon/9$ when z is sufficiently close to t.

As for I_1''', then when z is sufficiently close to t,

$$I_1'''\le C_9|\phi(t)-\phi(t_z)|=O(|t-t_z|^\nu)\le O(|t-z|^\nu)<\epsilon/9,$$

where C_9 is positive constant.

From the above, we have:

$$\lim_{\substack{z\to t\\ z\in D^\pm}}\int_\Omega[\phi(\xi)-\phi(t_z)]K(\xi,z)=\int_\Omega[\phi(\xi)-\phi(t)]K(\xi,t). \tag{10}$$

Hence by (7)–(10), we have:

$$\Phi^+(t)=\lim_{\substack{z\to t\\ z\in D^+}}\Phi(z)=\int_\Omega[\phi(\xi)-\phi(t)]K(\xi,t)+\phi(t), \tag{11}$$

$$\Phi^-(t)=\lim_{\substack{z\to t\\ z\in D^-}}\Phi(z)=\int_\Omega[\phi(\xi)-\phi(t)]K(\xi,t). \tag{12}$$

In addition, by (3)

$$\int_\Omega\phi(t)K(\xi,t)=\phi(t)\int_\Omega K(\xi,t)=\phi(t)\cdot\frac{\beta(t)}{s}. \tag{13}$$

Substituting (13) in (11) and (12), we get (1) and (2).

The above proof is obviously applicable to the case where Ω is piecewise smooth non-closed surface and t is not on the edge of Ω.

3. Boundary Properties of B-M Type Integral

Theorem 2 *If Ω is piecewise smooth closed surface, $\phi(\xi)$ satisfies Hölder condition with exponent ν on Ω, then for any $z\in D^+$ or $D^-, t\in\Omega$, we have*

$$|\Phi(z)-\Phi^\pm(t)|\le C_{10}|z-t|^\nu,\quad C_{10}=const>0, 0<\nu<1. \tag{14}$$

Proof Write $\Phi(z)$ as the form:

$$\Phi(z)=\int_{\Omega}[\phi(\xi)-\phi(t_z)]K(\xi,z)+\Phi(t_z)\int_{\Omega}K(\xi,z)=:\Phi_1(z)+\Phi_2(z).$$

Let $|z-t|$ be sufficiently small. Now draw a sphere $B_\delta(t)$ using t as the center, $\delta=2|z-t|$ as the radius, put $\omega_\delta=B_\delta(t)\cap\Omega$. For any $z\notin\Omega, t\in\Omega$, we have

$$\begin{aligned}|\Phi_1(z)-\Phi_1(t)| &\leq |\int_{\Omega-\omega_\delta}[\phi(\xi)-\phi(t_z)]K(\xi,z)-\int_{\Omega-\omega_\delta}[\phi(\xi)-\phi(t)]K(\xi,t)|\\ &\quad +|\int_{\omega_\delta}[\phi(\xi)-\phi(t_z)]K(\xi,z)|+|\int_{\omega_\delta}[\phi(\xi)-\phi(t)]K(\xi,t)|\\ &\leq |\int_{\Omega-\omega_\delta}[\phi(\xi)-\phi(t_z)][K(\xi,z)-K(\xi,t)]+|\int_{\Omega-\omega_\delta}[\phi(t_z)-\phi(t)]K(\xi,t)|\\ &\quad +|\int_{\omega_\delta}[\phi(\xi)-\phi(t_z)]k(\xi,z)|+|\int_{\omega_\delta}[\phi(\xi)-\phi(t)]K(\xi,t)|\\ &= :\Phi_{11}(z)+\Phi_{12}(z)+\Phi_{13}(z)+\Phi_{14}(z),\end{aligned}$$

where $t_z\in\omega_\delta$ and $|t_z-z|=\inf_{\xi\in\omega_\delta}|\xi-z|$. By Lemma 2, we have

$$\Phi_{13}(z)\leq C_{11}\delta^\nu=2^\nu C_{11}|z-t|^\nu,\quad \Phi_{14}(z)\leq 2^\nu C_{12}|z-t|^\nu,$$

where C_{11}, C_{12} are positive constants. Considering Φ_{12}, by (6) we get:

$$\Phi_{12}(z)\leq|\phi(t_z)-\phi(t)||\int_{\Omega-\omega_\delta}K(\xi,t)|\leq|\phi(t_z)-\phi(t)|\leq C_{13}|t-z|^\nu,$$

where C_{13} is positive constant. As for Φ_{11}, we have as I_1':

$$|K(\xi,z)-K(\xi,t)|<C_{14}\frac{|t-z|}{|\xi-t|^{2n}}d\sigma,$$

hence

$$\Phi_{11}(z)\leq C_{14}\int_{\Omega-\omega_\delta}\frac{|t-z|}{|\xi-t|^{2n}}|\phi(\xi)-\phi(t_z)|d\sigma\leq 2C_{14}|t-z|\int_{\Omega-\omega_\delta}\frac{d\sigma}{|\xi-t|^{2n-\nu}},$$

where C_{14} is positive constant. Using spherical coordinates we get:

$$\Phi_{11}(z)\leq C_{15}|t-z|\int_{\delta=2|t-z|}^{R}\rho^{\nu-2}d\rho\leq C_{16}|t-z|^\nu,$$

where C_{15}, C_{16} are positive constents, $R=\sup_{\xi\in\Omega}|\xi-t|$.

In short, there is $|\Phi_1(z)-\Phi_1(t)|\leq C|t-z|^\nu$.

On the other hand, for $z\in D^+, \Phi(z)=\phi(t_z)$,

$$\Phi_2^+(t)=\lim_{\substack{z\to t\\ z\in D^+}}\phi(t_z)\int_{\Omega}K(\xi,z)=\phi(t),$$

then by (6)

$$|\Phi_2(z) - \Phi_2^+(t)| = |\phi(t_z) - \phi(t)| \le C_3|t - z|^\nu, \quad t \in \Omega.$$

when z is close to another point t_0 on Ω from the inner of D^+, we have

$$|\Phi_2^+(t_0) - \Phi_2^+(t)| \le C_3|t - t_0|^\nu, \quad t_0, t \in \Omega.$$

For $z \in D^-, \Phi_2^-(t) = 0$, correspondingly there is

$$|\Phi_2(z) - \Phi_2^-(t)| = 0, \quad t \in \Omega.$$

From the above, we have got (14).

Notice that if Ω is smooth close surface, then by Plemelj formulla there is $\Phi(t) = \frac{1}{2}[\Phi^+(t) + \Phi^-(t)]$. So by theorem 2 we have the result that if $\phi(\xi)$ satisfies Hölder condition on Ω, then $\Phi(t)$ also satisfies Hölder condition. But if Ω is piecewise smooth closed surface (that is Ω has angular points and sharp points), the result is not right.

Since

$$\Phi(t) = \int_\Omega \phi(\xi)K(\xi, t) = \frac{1}{2}[\Phi^+(t) + \Phi^-(t)] + (1 - \frac{\beta(t)}{s})\phi(t),$$

where $\beta(t)$ is not a continuous function.

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关于多复变数 Cauchy 型积分的边界性质

陈 叔 瑾

(厦门大学研究生院,厦门 361005)

摘 要

本文用新的方法研究 B-M 型积分的边界性质,所得结果推进了文[1]的结果,并指出文[4]证明有错误.