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Note on Hardy-Littlewood Maximal Function *

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Hardy-Littlewood maximal function is defined as follows:

$$M(f)(x) = \sup_{x \in Q} \frac{1}{|Q|} \int_Q |f(y)| dy, \quad f \in L_{\text{loc}}(R^n),$$

where the supreum is taken over all cubes with sides parallel to the coordinate axes. The purpose of this note is to study the boundedness of $M(f)$ on the spaces bmo (bmo$\subset$BMO) and $\text{Lip}_\beta (0 < \beta < 1)$. The results are as follows:

Theorem 1 *If* $f \in$ bmo and $\inf_{x \in R^n} M(f)(x) < \infty$. *Then* $M(f)$ *is finite almost everywhere on* R^n, *and* $\|M(f)\|_{\text{bmo}} \le c\|f\|_{\text{bmo}}$.

Theorem 2 *If* $f \in \text{Lip}_\beta (0 < \beta < 1)$ *and* $\inf_{x \in R^n} M(f)(x) < \infty$, *then* $M(f)$ *is finite almost everywhere on* R^n. *Moreever we have* $\|M(f)\|_{\text{Lip}_\beta} \le c\|f\|_{\text{Lip}_\beta}$.

For the dyadic Hardy-Littlewood maximal function $M_d(f)$, we have:

Theorem 3 *If* $f \in$ BMO *and* $\inf_{x \in R^n} M_d(f)(x) < \infty$, *then* $M_d(f)$ *is finite almost everywhere on* R^n, *and* $\|M_d(f)\|_{\text{BMO}} \le c\|f\|_{\text{BMO}}$.

Theorem 4 *If* $f \in$ bmo *and* $\inf_{x \in R^n} M_d(f)(x) < \infty$, *then* $M_d(f)$ *is finite almost everywhere on* R^n. *Furthermore we have* $\|M_d(f)\|_{\text{bmo}} \le c\|f\|_{\text{bmo}}$.

Theorem 5 *If* $f \in \text{Lip}_\beta (0 < \beta < 1)$ *and* $\inf_{x \in R^n} M_d(f)(x) < \infty$, *then* $M_d(f)$ *is finite almost everywhere on* R^n. *Furthermore we have* $\|M_d(f)\|_{\text{Lip}_\beta} \le c\|f\|_{\text{Lip}_\beta}$.

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