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Preconditioning Method Based on Incomplete Decomposition for Nonsymmetric Systems*

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Vorst [2] discussed the incomplete decomposition of A or/and $A+A^T$ in combination with some iterations to solve a large nonsymmetric system of linear equations

$$Ax=b,\quad A=(a_{ij})\in R_n^{n\times n},\quad A\neq A^T. \tag{1}$$

Here we present an incomplete decomposition of A and construct a preconditioning method for solving (1). Assume that A is also a nonsingular M-matrix, then there hold

$$A=K-R,\quad K=(L+\bar{D})\bar{D}^{-1}(U+\bar{D}),\quad \mathrm{diag}(A)=\mathrm{diag}(K), \tag{2}$$

where L and U are strictly lower (upper) triangular parts of A, respectively. If $\bar{D}^{-1}>0$, applying it for preconditioning (1), we have the equivalent system $\bar{A}x=\bar{b}$, with $\bar{A}=\bar{D}^{-1}A, \bar{b}=\bar{D}^{-1}b$. Here $\bar{A}$ is also an M-matrix, hence we have

$$\bar{A}=(\bar{L}+I)(\bar{U}+I)-\bar{R},\quad \bar{L}=\bar{D}^{-1}L,\quad \bar{U}=\bar{D}^{-1}U. \tag{3}$$

Take $(\bar{L}+I)^{-1}$ as a left preconditioner for $\bar{A}x=\bar{b}$, we have the preconditioned system

$$\hat{A}\hat{x}=\hat{b},\quad \hat{A}=(\bar{L}+I)^{-1}\bar{A}(\bar{U}+I)^{-1},\quad \hat{x}=(\bar{U}+I)x,\quad \hat{b}=(\bar{L}+I)^{-1}b. \tag{4}$$

Theorem *Assume that both A and $A+A^T$ are nonsingular M-matrices, then all eigenvalues of $\hat{A}$ lie in a circle with centre (1,0) and radius less than 1.*

The conclusion of the theorem is more specific than that of [1,2] and will still be valid if A is a nonsingular M-matrix and $\bar{D}^T>0$.

Corollary *GCR(k) and the Chebyshev iteration for (2) converge to the solution of (1).*

References

[1] H.A.Vander Vorst (1981), J. of Comp. Physics, 1–19.
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