Therefore $c = 0$, which implies that μ is an invariant measure to T_t, and by the conclusion of [6] we have $\mu(dx) = \alpha m(dx)$ for α being a positive constant.

Thus, the proof of theorem 3.2 has been completed.

Acknowledgement *The author is gratefull to Prof. Wang Zi-kun and Prof. Wu Rong for their encouragement. He also thanks all members at stockastic process seminar in Nankai University for their helpful suggestions.*

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一类测度值分支过程的不变特性

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摘 要

空间齐次性是 R^d 上 Lévy 过程的一个重要特性，本文考虑超 Lévy 过程的类似性质，即是分布意义下的平移不变性，并且对一类特殊的测度值分支过程当其初始测度是 Lebesgue 测度时，得到了更强的结果.

Journal of Mathematical Research & Exposition
Vol.15, No.4, 509-516, Nov. 1995

The Invariant Characters of One Class of Measure-valued Branching Processes *

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Abstract The spatial homogeneity is an important character for Lévy-processes on Euclidean space R^d. The analogy for the Measure-valued branchinng processes over Lévy processes (also called Super-Lévy processes) will be discussed in this paper. That is the translation-invariant in the sense of distributions. Moreover some stronger results are also derived for the special Measure-valued branching processes when their initial measures are given by the Lebesgue measure.

Keywords measure-valued branching processes, translation- invariant, invariant measure, relatively invariant processes.

Classification AMS(1991) 60J25,60G80/CCL O211.62

1 Introduction

The measure-valued branching process (MBP) arises as the high density limit of a certain branching particle system. The general MBP have been constructed by El Karoui and Roelly-Coppoletta[1], Fitzsimmons[2] and so on. In this paper we follow the definition of El Karoui and Roelley-Coppoletta.

Firstly, we introduce some notations:

Let C_b denote all continuous functions on R^d, and C_0^∞ be the space of functions on R^d having infinite-order derivatives and vanishing at infinity, $C_c^\infty(R^d)$ denote all continuous function having compact support and infinite-order derivations. $M(R^d)$ denotes the space of all Radon measures on R^d. We call $g_p(x) = \frac{1}{(1+|x|^2)^{p/2}}(p > d)$ a reference function and set

$$M_p(R^d) = \{\mu \in M(R^d) : (1+x^2)^{-p/2}\mu(dx) \text{ is a finite measure}\},$$

$$C_p(R^d) = \{f \in C(R^d) : \|f/g_p\|_{\max} < \infty\},$$

for $f \in C_C^\infty(R^d)$, we also define $\langle \mu, f\rangle = \int_{R^d} f(x)\mu(dx)$.

For notations not defined here refer to [1].

We now turn to Meauser-valued branching processes.

*Received Nov.20, 1993.

Let A be the strong generator of a Feller semigroup T_t on $C(R^d)$ with domain $D(A)$. Let $\Psi(x,\cdot)$ be a branching function defined by

$$\Psi(x,z) = a(x) + b(x)z - c(x)/2z^2 - \int_{(0,\infty)} (e^{-z\lambda} - 1 + z\lambda I_{\{0<\lambda\leq 1\}})\nu(x,d\lambda)$$

with the coefficients satisfying the following conditions:

$a(x), b(x), c(x)$ belong to $C_b(R^d)$, and $a(x), b(x)$ are non-negative. The measure ν satisfies $\int_{(0,\infty)}(\lambda^2 \wedge 1)\nu(x,d\lambda) < \infty$ and is sufficiently regular on x to get $z \to R(x,z)$ being a continuous function uniformly on x.

Based on A and Ψ, a Measure-valued branching process X_t on $M_p(R^d)$ has been constructed by El Karoui and Roelley-Coppoletta[1], and the Laplace functional of X_t can be expressed as follows

$$E_\mu \exp\{-\langle X_t, f\rangle\} = \exp\{-\langle \mu, V_t f\rangle\}, \tag{1.1}$$

where $V_t f$ satisfies the equation or

$$V_t f - \int_0^t T_{t-s}(\Psi(V_s f(x)))ds = T_t f,\ f \in C_0^\infty(R^d)^+,$$

$$\frac{\partial V_t f}{\partial t} = AV_t f + \Psi(V_t f), \quad V_0 f = f.$$

The purpose of this paper is to find some invariant characters for the Measure-valued branching processes when their originial processes are spatially homogeuous processes on R^d. After introducing the MBP and giving some notations in the first section, we shall discuss, in section 2, the invariant character in the sense of distributions for the Measure-valued branching Lévy processes (Super-Lévy processes). and at the end of section 2, we also derive a stronger result for special MBP. In section 3 we shall show a uniqueness theorem for the relatively invariant Measure-valued branching Brownian motion (defined in section 3) and extend it to Measure- valued branching diffusion processes.

In what follows, we assume that the branching functions $\Psi(z) = bz - c/2z^2$, (b and c are two constant numbers, $c > 0$).

2. The translation-invariant character for Measure-valued branching Lévy processes

Theorem 2.1 *Let X_t be a measure-valued branching Lévy process with branching function $\Psi(z) = bz - c/2z^2$, and the initial measure be the Lebesgue measure on R^d. Then X_t is translation-invariant in the sense of distribution w.r.t P_m, i.e.,*

$$X_t(A) \stackrel{d}{=} X_t(A+y), \quad \forall A \in B(R^d), \forall y \in R^d$$

Proof (1.1) implies that

$$V_t f(x) - \int_0^t \int_{R^d} p(t-s,x,z)\Psi(V_s f(z))dzds = T_t f(x). \tag{2.2}$$

As we know that there is a unique positive solution to (1.1), also to (2.2)[2].

If replacing x by $x+y$ in (2.2), we have

$$(V_tf)(x+y) - \int_0^1 \int_{R^d} p(t-s, x+y, z)\Psi(V_s(f(z))dzds = (T_tf)(x+y). \qquad (2.3)$$

On the other hand, if using $f(x+y)$ instead of $f(x)$ in (2.2), then we obtain

$$V_tf(\cdot+y) - \int_0^t \int_{R^d} p(t-s, x, z)\Psi(V_sf(z+y))dzds = T_tf(\cdot+y). \qquad (2.4)$$

Noting that $p(t, x+y, z) = p(t, x, z-y)$, hence we have

$$(T_tf)(x+y) = \int_{R^d} p(t, x+y, z)f(z)dz = \int_{R^d} p(t, x, z-y)f(z)dz = T_tf(\cdot+y)$$

and

$$\int_{R^d} p(t-s, x+y, z)\Psi(V_sf(z))dz = \int_{R^d} p(t-s, x, z)\Psi((V_sf)(z+y))dz.$$

Thus is follows from (2.3) that

$$(V_tf)(x+y) - \int_0^t \int_{R^d} p(t-s, x, z)\Psi((V_sf)(z+y))dzds = T_tf(x+y). \qquad (2.5)$$

By contrasting (2.5) with (2.4), and using the uniqueness of the solution of (2.2) we have

$$(V_tf)(x+y) = V_t(f(\cdot+y))(x)$$

for any $y \in R^d, t > 0$.

Recall the equation (1.1), we also have

$$\begin{aligned} E_m \exp\{-\langle X_t(\cdot), f(\cdot+y)\rangle\} &= \exp\{-\langle m(\cdot), (V_tf(\cdot+y)\rangle\} = \exp\{-\langle m(\cdot), (V_tf)(\cdot+y)\rangle\} \\ &= \exp\{-\langle m(\cdot), V_tf(\cdot)\rangle\} = E_m \exp\{-\langle X_t(\cdot), f(\cdot)\rangle\} \end{aligned}$$

which implies

$$E_m \exp\{-\langle X_t(\cdot+y), f(\cdot)\rangle\} = E_m \exp\{-\langle X_t(\cdot), f(\cdot)\rangle\}.$$

Hence $X_t(\cdot+y) \stackrel{d}{=} X_t(\cdot), P_m, \forall y \in R^d$ and by the arbitrarity of f, this also implies

$$X_t(A) \stackrel{d}{=} X_t(A+y), P_m, \quad \forall A \subset R^d, \forall y \in R^d.$$

Thus we complete the proof of the theorem. □

Based on theorem 2.1, we try to develop the results in spacial cases. To do so, we need the following lemma whose proof was essentially due to Konnô and Shiga[5].

Lemma 2.1 *Let X_t be a Super-symmetric Lévy process with branching functions $\Psi(z) = bz - \frac{c}{2}z^2$, and suppose that $\int_0^T p(s,0)ds < \infty, \quad \forall T > 0, P(X_0 = m) = 1$. Then X_t is absolutely continuous with respect to the Lebesgue measure m.*

Proof Applying Fitzsimmon's results [3] of second-order moments of superprocesses, we have

$$E_\mu\langle X_t, f\rangle^2 = \langle\mu(\cdot), E(\exp\{\int_0^b b(\xi_s)ds\}f(\xi_t))\rangle^2 + \int_0^t \langle\mu P_{t-s}(\cdot), c(\cdot)(E(\exp\{\int_0^s b(\xi_r)dr\}f(\xi_s)))^2\rangle ds.$$

If $b(z)\equiv b, c(z)\equiv c, \mu = m$, then

$$E_m\langle X_t, f\rangle^2 = \exp\{2bt\}\langle m, T_t f\rangle^2 + c\int_0^t \langle mT_{t-s}, \exp\{2bs\}(T_s f)^2\rangle ds = \exp\{2bt\}\langle m, f\rangle^2 + c\int_0^t \exp\{2bs\}\langle m, (T_s f)^2\rangle ds.$$

On the other hand, let $X_t^h(x)\equiv\langle X_t(\cdot), p(h,x,\cdot)\rangle$, we have that

$$\begin{aligned}&\int_{R^d} E_m(X_t^h(x))^2 g_p(x)dx \\ &= \int_{R^d}\exp\{2bt\}g_p(x)dx + c\int_{R^d} g_p(x)\int_0^t \exp\{2bs\}\langle m, (T_s ph)^2\rangle dsdx \\ &= \exp\{2bt\}\int_{R^d}\frac{1}{(1+x^2)^{p/2}}dx + c\int_0^t \exp\{2bs\}p_{2s+2h}(o)ds\int_{R^d} g_p(x)ds \\ &\le \exp\{2bt\}(1 + c\int_{2h}^{2t+2h} p_s(o)ds)\int_{R^d}\frac{1}{(1+x^2)^{p/2}}dx < +\infty\ (p>d),\end{aligned}$$

and

$$\begin{aligned}&\int_{R^d} E_m(X_t^{h_1}(x) - X_t^{h_2}(x))^2 g_p(x)dx \\ &= \int_{R^d}\cdot c\int_0^t \exp\{2bs\}\int_{R^d}[p_{s+h_1}(x,y) - p_{s+h_2}(x,y)]^2 dyds g_p(x)dx \\ &= c\int_{R^d}\int_0^t \exp\{2bs\}[p_{2s+2h_1}(0) - 2p_{2s+h_1+h_2}(0) + p_{2s+2h_2}(0)]ds g_p(x)dx \\ &\le \int_{R^d} g_p(x)dx\exp\{2bs\}[(\int_{2h_1}^{h_1+h_2} - \int_{h_1+h_2}^{2h_2} + \int_{2t+h_1+h_2}^{2t+2h_2} - \int_{2t+2h_2}^{2t+h_1+h_2})p(s,0)ds] \\ &\quad (h_2 > h_1 > 0) \\ &\to 0(h_1, h_2\downarrow 0).\end{aligned}$$

(by the integrablity of $p(s,0)$ on $[0,T]$).

Therefore there is an $X_t(x)$ such that

$$\int_{R^d} E_m(X_t^h(x) - X_t(x))^2 g_p(x)dx \to 0, \text{ when } h\to 0.$$

But for any $\phi\in C_C^\infty(R^d)$, $(X_t^h,\phi) = \int_{R^d} X_t^h(x)\phi(x)dx \to \langle X_t,\phi\rangle(h\to 0)$, so we have

$$\begin{aligned}&E_m|\langle X_t,\phi\rangle - \int_{R^d} X_t(x)\phi(x)dx|^2 = E_m(\lim_{h\downarrow 0}|\langle X_t^h,\phi\rangle - \int_{R^d} X_t(x)\phi(x)dx|) \\ &\le \lim_{h\downarrow 0}\int_{R^d} E_m|X_t^h(x) - X_t(x)|^2 g_p(x)dx\int_{R^d} g_p^{-1}(x)(\phi(x))^2 dx \to 0(h\downarrow 0),\end{aligned}$$

namely $\langle X_t, \phi\rangle = \int_{R^d} X_t(x)\phi(x)dx$ a.s., P_m.

Remark 2.1 When $d = 1$, Super-Brownian motion and Super α- symmetric stable process $(1 < \alpha < 2)$ satisfy the conditions of Lemma 2.1.

Particularly, let $c = 0$ in lemma 2.2 we know that V_t in (1.1) is linear, hence

$$\begin{aligned}
&E_m|\exp\{-\langle X_t(\cdot), f(\cdot+y)\rangle\} - \exp\{-\langle X_t(\cdot), f(\cdot)\rangle\}|^2\\
&= E_m\exp\{-\langle X_t(\cdot), 2f(\cdot+y)\rangle\} - 2E_m\exp\{-\langle X_t(\cdot), f(\cdot+y)+f(\cdot)\rangle\}\\
&\quad + E_m\exp\{-\langle X_t(\cdot), 2f(\cdot)\rangle\}\\
&= \exp\{-\langle m, V_t(2f(\cdot+y))\rangle\} - 2\exp\{-\langle m, V_t(f((\cdot+y)+f(\cdot))\rangle\}\\
&\quad + \exp\{-\langle m, V_t(2f(\cdot))\rangle\}\\
&= 2\exp\{-\langle m, V_t(2f)\rangle\} - 2\exp\{-\langle m, 2V_tf\rangle\} = 0,
\end{aligned}$$

namely

$$\langle X_t(\cdot), f(\cdot+y)\rangle = \langle X_t(\cdot), f(\cdot+y)\rangle = \langle X_t(\cdot), f(\cdot)\rangle, \text{ a.s., } P_m. \tag{2.6}$$

It seems that the exceptional set in (2.6) depends on f and y, but as a matter of fact, it does not. This could be explained as follows.

Suppose $S = \{y_n, n > 0\}$ be the set of all rational points in R^d. For fixed $f \in C_c^\infty(R^d)$, we have

$$\langle X_t(\cdot), f(\cdot+y)\rangle = \langle X_t(\cdot), f(\cdot)\rangle \forall y \in S \text{ on } N_1, P_m(N_1) = 1$$

and if $y_0 \notin S$,

$$\langle X_t(\cdot), f(\cdot+y_1)\rangle = \lim_{y_n\to y_0}\langle X_t(\cdot), f(\cdot+y_n)\rangle = \langle X_t(\cdot), f(\cdot)\rangle$$

by the continuity of f.

Moreover, from the separablity of the space $C_c^\infty(R^d)$ we can deduce by the same way as above that, on $N_2, \langle X_t, f(\cdot+y)\rangle = \langle X_t, f(\cdot)\rangle$, for any $f \in C_c^\infty(R^d)$, where $P_m(N_2) = 1$.

Therefore for any $y \in R^d$ and any $f \in C_c^\infty$, on $N_1 \cap N_2$, we get

$$\langle X_t, f(\cdot+y)\rangle = \langle X_t, f(\cdot)\rangle$$

in addition, $\langle X_t, f\rangle = \int_{R^d} X_t(x)f(x)dx$, so we have, on $N = N_1 \cap N_2$, for any y and f

$$\int_{R^d} X_t(x)f(x)dx = \int_{R^d} X_t(x)f(x+y)dx = \int_{R^d} X_t(x-y)f(x)dx,$$

i.e., $\int_{R^d}[X_t(x) - X_t(x-y)]f(x)dx = 0$.

From the arbitrarity of $y \in R^d$ and $f \in C_C^\infty$, it follows that

$$X_t(x_1) = X_t(x_2), \text{ a.e., } x_1, x_2 \in R^d.$$

Hence there is a random variable c_t taking values in R such that

$$X_t(dx) = c_t m(dx) \text{ for } t > 0.$$

Thus we obtain

Proposition 2.1 *Let X_t be a Measure-valued branching Lévy process with branching function $\Psi(z) = bz$, and assume that the conditions in theorem 2.1 and Lemma 2.2 hold. Then*

$$P_m(X_t = c_t m) = 1, \text{ for fixed } t > 0, \tag{2.7}$$

where c_t is a ramdom variable taking values in R^+.

The Measure-valued branching process satisfying the last identity is called a relatively invariant process.

3. The uniqueness theorems of the initial measures for the relatively invariant processes

Motivated by the uniqueness problem of invariant measure for Markov process on R^d, we shall consider the similar problem for a relatively invariant measure-valued branching process in this section. Now we give our main results.

Theorem 3.1 *Let X_t be a Measure-valued branching Brownian motion with branching function $\Psi(z) = bz - c/2z^2$. If the process is relatively invariant with the initial measure μ. Then $\mu = \alpha m$, (α is a positive constant, m is the Lebesgue measure on R^d).*

Furthermore, we can extend this conclusion to a class of Measure-valued branching diffiusion processes. That is

Theorem 3.2 *Suppose that*

1. *ξ_t is an L-diffusion process with transition density $p(t, x, y)$ satisfying*

$$\frac{1}{\nu_3(2\pi t)^{d/2}} e^{-\frac{\nu_4|x-y|^2}{2t}} \leq p(t,x,y) \leq \frac{1}{\nu_1(2\pi t)^{d/2}} e^{-\frac{\nu_2|x-y|^2}{2t}} \quad (\nu_1, \nu_2, \nu_3, \nu_4 > 0).$$

2. *X_t is a Measure-valued branching process on $M_p(R^d)$ over ξ_t with the branching function $\Psi(z) = bz - \frac{1}{2}cz^2$.*

3. *$P_\mu(X_t = c_t\mu) = 1, \forall t > 0, \mu \in M_p(R^d)$, for fixed t, c_t is a random variable taking values in R.*

Then $\mu = \alpha m$(α is a constant, m is the Lebesgue measure on R^d).

Remark 3.1 To prove the theorem, we need an analytic fact: Let $\alpha(t), (t > 0$ be a continuous function, and

$$\alpha(t_1 + t_2) = \alpha(t_1)\alpha(t_2), \alpha(t) > 0.$$

Then there exists a positive c, such that $\alpha(t) = e^{-ct}$.

Proof of theorem 3.2 According to Fitzsimmon[1], we know that

1. X_t is a continuous process in the topology of vague convergence.
2. $E_\mu\langle X_t, f\rangle = \langle\mu, e^{-bt}T_t f\rangle, \forall f \in C_c^\infty(R^d)$.

Hence, by the assumption 3 of theorem 3.2, we have

$$\langle\mu, e^{-bt}T_t f\rangle = E_\mu\langle c_t\mu, f\rangle = E_\mu(c_t)\langle\mu, f\rangle.$$

Let $\alpha(t)=e^{bt}E_{\mu}(c_t)$, and suppose that $u\not\equiv 0$, then we can take a positive function $f\in C_C^{\infty}$ such that $\langle\mu,f\rangle>0$.

Since $T_tf(x)$ is continuous in t, we can take two positive number t_0 and δ satisfying $t_0-\delta>0$, by assumption 1 of the theorem, we are easy to find that for $N>0$ large enough and $t\in(t_0-\delta,t_0+\delta)$

$$\begin{aligned} T_tf(x) &\le \|f\|\int_{\mathrm{supp}f}p(t,x,y)dy\le m(\mathrm{supp}(f))\|f\|c\frac{1}{(1+|x|^2)^{p/2}}\ (|x|\ge N)\\ T_tf(x) &\le \|f\|,\ (x\le N). \end{aligned}$$

Since $\langle\mu,\frac{1}{(1+|x|^2)^{p/2}}\rangle<\infty$, then it follows from the dominated convergence theorem that $\langle\mu,T_tf(x)\rangle$ is continuous in $(t_0-\delta,t_0+\delta)$, and so in $(0,\infty)$, which also implies that $\alpha(t)$ is continuous on $(0,\infty)$. Besides, $\mu T_t=\alpha(t)\mu$ (clear $\alpha(t)>0$) and

$$\alpha(t_1+t_2)=\mu T_{t_1+t_2}=(\mu T_{t_1})T_{t_2}=\alpha(t_1)\mu T_{t_2}=\alpha(t_1)\alpha(t_2),$$

so we conclude that $\alpha(t)=e^{-ct}$ by lemma 2.2.

In what follows, we intend to prove $c=0$.

If $c\notin 0$, we consider the following two cases.

1. Assume $c>0$, we choose a bounded $A\in B(R)$ with $\mu(A)>0$, then

$$\begin{aligned} \mu(A) &= e^{nct}\mu T_{nt}(A)\ge e^{nct}\int_{R^d}\mu(dx)\int_A p(nt,x,y)dy\\ &\ge e^{nct}\int_{R^d}\mu(dx)\int_A\frac{1}{\nu_3(2\pi nt)^{d/2}}e^{-\frac{\nu_1|x-y|^2}{2nt}}dy\\ &\ge e^{nct}m(A)\inf_{y\in A}\frac{1}{\nu_3(2\pi nt)^{d/2}}e^{-\frac{\nu_1|y|^2}{2nt}}\int_{R^d}e^{-\frac{\nu_1|x|^2}{2nt}}\mu(dx)\\ &\ge m(A)\inf_{y\in A}e^{-\frac{\nu_1|y|^2}{2nt}}\frac{e^{\frac{1}{2}nct}}{\nu_3(2\pi nt)^{d/2}}\int_A e^{-\frac{\nu_1|x|^2}{2nt}+\frac{1}{2}nct}\mu(dx)\\ &\to \infty(n\uparrow+\infty) \end{aligned}$$

2. Assume $c<0$, we let $\mu(A)>0$, then

$$\begin{aligned} \mu(A) &= \frac{\mu T_{nt}(A)}{e^{n|c|t}}\\ &\le \frac{1}{e^{n|c|t}}\int_{R^d}\mu(dx)\int_A\frac{e^{-\nu_2\frac{|x-y|^2}{2nt}}}{\nu_1(2\pi nt)^{d/2}}dy\\ &\le m(A)\sup_{y\in A}e^{\nu_2\frac{|y|^2}{2nt}}\frac{e^{-n|c|t}}{\nu_1(2\pi nt)^{d/2}}\int_{R^d}e^{-\nu_2\frac{|x|^2}{2nt}}\mu(dx)\\ &\le m(A)\sup_{y\in A}e^{\nu_2\frac{|y|^2}{2nt}}\frac{e^{-n|c|t}}{\nu_1(2\pi nt)^{d/2}}\int_{R^d}\frac{\mu(dx)}{1+\nu_2\frac{|x|^2}{2nt}+\cdots+\frac{1}{[p]!}(\nu\frac{|x|^2}{2nt})^{[p]}}\\ &\to 0(n\uparrow+\infty) \end{aligned}$$

which contradicts with $\mu(A)>0$.

Therefore $c = 0$, which implies that μ is an invariant measure to T_t, and by the conclusion of [6] we have $\mu(dx) = \alpha m(dx)$ for α being a positive constant.

Thus, the proof of theorem 3.2 has been completed.

Acknowledgement *The author is gratefull to Prof. Wang Zi-kun and Prof. Wu Rong for their encouragement. He also thanks all members at stockastic process seminar in Nankai University for their helpful suggestions.*

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一类测度值分支过程的不变特性

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摘 要

空间齐次性是 R^d 上 Lévy 过程的一个重要特性，本文考虑超 Lévy 过程的类似性质，即是分布意义下的平移不变性，并且对一类特殊的测度值分支过程当其初始测度是 Lebesgue 测度时，得到了更强的结果.