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Some Conclusions on Categories with Terminal Objects *

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Abstract In this paper, some conclusions are discussed for the categories with terminal objects, such as some results under conditions satisfied Axiom (P) and Axiom (U), and some propositions on image and inverse image are discussed and proved also.

Keywords image, pullback, union, quasiregular.

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0. Introduction

The category $C - c^r$, which is defined in [1] for describing a changing process of a system, possesses a terminal object and has finite products. There are many categories with terminal objects which relate to abelian category in varying degrees, such as Set (the category of sets), P/X (the category of presheaves of sets on a topological space X), S/X (the category of sheaves of sets) and Toposes. We observe that the additive operation in an abelian category creates a null object. Can we find, within the scope of HAA, anything substantially independent of null objects and extend it to some categories with terminal objects? Where HAA means homological algebra in abelian categories.

Since an abelian group is a $\{0, -, +\}$-algebra (see [2]), can we establish an approach to HAA for the category Ω-Alg with Ω a family of operations? Moreover, since the category AG of abelian groups is a special one of the categories AG_{r_n} of commutative n-groups, $AG = AG_{r_2}$, can we put forward a united theory which is HAA when $n = 2$? Both Ω-Alg and $AG_{r_n}(n \geq 2)$ have terminal objects.

To answer the above metioned problems, we defined the quasikernels for the categories with terminal objects as follows (see [3]):

Let F be a terminal object. If the diagram $\begin{tikzcd} \bullet \arrow[r, "s"] \arrow[d, "u"'] & F \arrow[d, "h"] \\ {} \arrow[r, "f"'] & {} \end{tikzcd}$ is a pullback, then the morphism u is called a (teminal) quasikernel of f and h is called a lateral of u. We write $QK^F(f)$ for the class of all quasikernels of f (It its also useful to the next paper).

A terminal object F is called to be quasinull, if for any object A the morphism $t : A \to F$ is epi.

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In this paper, the symbols are the same as those in [4], F always denotes a terminal object. 1 is an identity morphism, $\rightarrowtail$ denotes a monic, $\twoheadrightarrow$ denotes a regular epi (i.e., it is a coequalizer of some pair of morphisms). If u is monic, $\langle u\rangle$ denotes the class $\{v \mid \mathrm{codom}(v) = \mathrm{codom}(u)$ and $v \cong u\}$. The composite of by a is denoted as $ba, (b)a$, or $b\cdot(a)$, but $b(a)$ denotes the image of a by b (see [5]). If there is no danger of confusion. u is used to replace $\langle u\rangle$, and we write (A, B) for the class of all the morphisms from A to B. The symbol $(a, b; c, d)$ means the diagram $\begin{tikzcd} \bullet \arrow[r, "d"] \arrow[d, "c"'] & \bullet \arrow[d, "b"] \\ \bullet \arrow[r, "a"'] & \bullet \end{tikzcd}$. It is often convenient that a subobject of a object A is called the subobject of a morphism $f: A \to B$.

§1. Axioms and Lemmas

A category is called to be quasireqular, if it satisfies the following

Axiom (QR) Every morphism $\alpha : A \to B$ can be written as a composition $A \twoheadrightarrow C \rightarrowtail B$; moreover for every diagram of the form $\begin{tikzcd}[row sep=tiny] \bullet \arrow[dr] & \\ & \bullet \\ \bullet \arrow[ur] & \end{tikzcd}$ there is a pullback diagram which is of the form $\begin{tikzcd}[row sep=tiny] & \bullet \arrow[dl] \arrow[dr] & \\ \bullet \arrow[dr] & & \bullet \\ & \bullet \arrow[ur] & \end{tikzcd}$.

A category is called to be weak regular, if it only satisfies the second part of the axiom (QR) (see [6, Axiom (EX1)]).

In the category AG of abelian groups, for any pair (f, g) of morphisms such that $\begin{tikzcd} & \bullet \arrow[d, "g"] \\ \bullet \arrow[r, "f"'] & \bullet \end{tikzcd}$, there is a pullback $\begin{tikzcd} \bullet \arrow[r, "b"] \arrow[d, "a"'] & \bullet \arrow[d, "g"] \\ \bullet \arrow[r, "f"'] & \bullet \end{tikzcd}$. In the proof of this proposition we used the fact $\mathrm{Im}(f) \cap \mathrm{Im}(g) \neq \emptyset$. In this category $\mathrm{Im}(f) \cap \mathrm{Im}(g) \neq \emptyset$ is equivalent to that there is a commutative diagram $\begin{tikzcd} \bullet \arrow[r] \arrow[d] & \bullet \arrow[d, "g"] \\ \bullet \arrow[r, "f"'] & \bullet \end{tikzcd}$, so we can replace the above fact by the following axiom:

Axiom (P) If there is a commutative diagram $\begin{tikzcd} \bullet \arrow[r, "v"] \arrow[d, "u"'] & \bullet \arrow[d, "g"] \\ \bullet \arrow[r, "f"'] & \bullet \end{tikzcd}$ with g a monic, then there is a pullback $\begin{tikzcd} \bullet \arrow[r, "b"] \arrow[d, "a"'] & \bullet \arrow[d, "g"] \\ \bullet \arrow[r, "f"'] & \bullet \end{tikzcd}$.

In Set and many categories based upon sets, the following axiom is a basic fact:

Axiom (U) If $f : A \twoheadrightarrow B$ is a regular epi and a_i is a subobject of $B, i \in T$, such that $\tilde{\cup}_T a_i = \langle 1\rangle$, then $\tilde{\cup}_T f^{-1}(a_i) = \langle 1\rangle$.

The meaning of $\tilde{\cup}$ will be shown below.

Axiom (U) is abstracted from the definition of a map in Set, let $f : A \to B$, then $f(a)$ takes sense for each $a \in A$.

A regular epi is called to be (U)-epi, if for it Axiom (U) holds; A category is called a (U)-category, if every regular epi in it is (U)-epi; For a quasiregular category, a morphism f is called a (U)-morphism, if the regular epi e, which is in the regular factorization $f = me$, is (U)-epi.

A category is called a (P)-category, if for it Axiom(P) holds.

We are going further into the meanings of Axiom(U) and Axiom(P) in §2.

For a category based upon sets, it is easy to demonstrate the following fact:

If every inclusion is a morphism of the category, then when $A_i \subset A$ is a subset of the object A and an object of the category, $i \in T, \cup_T A_i = A \Leftrightarrow \cup_T A_i$ is an object of the category and $\tilde{\cup}_T I_{A_i} = \langle 1_A \rangle$. Where $\cup_T A_i$ is a set union, $\tilde{\cup}_T I_{A_i}$ is a morphism union, where $I_{A_i} : A_i \to A$ is the inclusion.

The above fact is useful when we research into the relation between $\cup A_i = A$ and $\tilde{\cup} I_{A_i} = \langle 1_A \rangle$, and the relation is used when we wonder whether a category subject to Axiom(U),

There are many important categories which are (P) (U)-quasiregular categories with terminal objects. such as Set, $P/X, S/X$, Top(I) (see [7]), some subcategories of $C - c^r$.

There are many (P) (U)-quasiregular categories with quasinull terminal objects, such as all the above mentioned categories but Top(I).

The conormalness plays an important role in HAA, in the present case, first of all, we have to make the quasi-conormalness clear.

Definition *For a category with a terminal object F, a epi $f : A \to B$ is called a (terminal) coquasikernel with any lateral, if for every $h \in (F, B)$ there is a morphism u such that the diagram* $\begin{array}{ccc} \bullet & \to & F \\ {\scriptstyle u}\downarrow & & \downarrow{\scriptstyle h} \\ \bullet & \xrightarrow[f]{} & \bullet \end{array}$ *is a pushout.*

There is no difficulty to prove that AG_{r_n} and ${}_RM_n^l (n \geq 2)$ are (P)-regular categories with quasinull terminal objects and subject to the following axiom:

Axiom (CQN) Every regular epi is a (terminal) coquasikernel with any lateral.

For AG_{r_n} (see [8]) if there is a diagram $\begin{array}{ccc} & & F \\ & & \downarrow \\ \bullet & \xrightarrow[f]{} & \bullet \end{array}$ with f an epi, then f is a coquasikernel; An epi in ${}_RM_n^l$ is always a (terminal) coquasikernel of a morphism, because $(F, M) \neq \emptyset$ for any $M \in \mathrm{ob}_R M_n^l$ (see [9]).

B.Mitchell's union of a family $\{a_i\}_{i \in T}$ of subobjects $a_i's$ is denoted by $\cup_T a_i$ (see [5, p.11]). If $a_i \leq a$, and if when a_i is carried into h by 1, a is also carried into h by 1, then we call a a weak-union of $\{a_i\}$. Clearly, if it exists, then it is unique up to an isomorphism, and we write $\tilde{\cup}_T a_i$ for it. Obviously, if $\cup a_i$ exists, then $\tilde{\cup} a_i$ exists and $\tilde{\cup} a_i = \cup a_i$.

Lemma 1.1 *If $\mathcal{A}$ is a quasiregular category satisfying Axiom(P), then $\mathcal{A}$ satisfies the following condition: For any commutative diagram $(a, b; c, d)$, there exists a pullback $(a, b; c', d')$.*

Lemma 1.2 *For any category with a terminal object, a quasikernel is an h-lateral quasikernel of its h-lateral coquasikernel and an h-lateral coquasikernel is an h-lateral coquasikernel of its h-lateral quasikernel.*

Lemma 1.3 *For a (P)-category, $\cup a_i = \tilde{\cup} a_i$ when $\tilde{\cup} a_i$ exists.*

Proof Suppose a_i is carried into a monic h by f. Then there is a pullback $(f, h; m, n)$ by Axiom(P), so that for each a_i there is a unique t_i such that $mt_i = a_i$, hence a_i is carried into m by 1. Therefore, there is morphism t such that $mt = \tilde{\cup} a_i$. We have $h \cdot (nt) = (hn)t = (fm)t = f \cdot (mt) = f \cdot (\tilde{\cup} a_i)$, this means that $\tilde{\cup} a_i$ is also carried into h by f, so that $\tilde{\cup} a_i = \cup a_i$. □

Lemma 1.4 *For any category $\mathcal{A}$, $\tilde{\cup} a_i = \langle 1 \rangle \Leftrightarrow$ every a_i is carried into h by 1 implies that h is an isomorphism.*

Lemma 1.5 *For any category, if $\tilde{\cup} a_i$ exists and $(\tilde{\cup} a_i)t_i = a_i$, then $\tilde{\cup} t_i = \langle 1 \rangle$.*

Proof At first, we demonstrate that if t_i is carried into h by $\tilde{\cup} a_i$ then $\tilde{\cup} a_i$ is carried into h by 1. In fact, t_i is carried into h by $\tilde{\cup} a_i$ implies that there is a morphism m_i such that $hm_i = (\tilde{\cup} a_i)t_i$. Hence $hm_i = a_i$, this means that a_i is carried into h by 1, so that $\tilde{\cup} a_i$ is also carried into h by 1.

Suppose t_i is carried into h by 1, that is, there is a morphism s_i such that $\bar{h} s_i = t_i$. Hence $(\tilde{\cup} a_i)t_i = (\tilde{\cup} a_i)\bar{h} s_i$, that is, t_i is carried into $(\tilde{\cup} a_i)h$ by $\tilde{\cup} a_i$. By the above statement, $\tilde{\cup} a_i$ is carried into $(\tilde{\cup} a_i)\bar{h}$ by 1, so there is a morphism m such that $((\tilde{\cup} a_i)\bar{h})m = \tilde{\cup} a_i = (\tilde{\cup} a_i)1$. On the other hand, $\bar{h}$ and $\tilde{U} a_i$ are monic, so that the diagram $(\tilde{\cup} a_i, (\tilde{\cup} a_i)\bar{h}; \bar{h}, 1)$ is a pullback. Hence there is s unique a such that $\bar{h}s = 1$, hence $\bar{h}$ is a monic retraction and so that $\bar{h}$ is an isomorphism. Therefore $\tilde{\cup} t_i = \langle 1 \rangle$ by Lemma 1.4. □

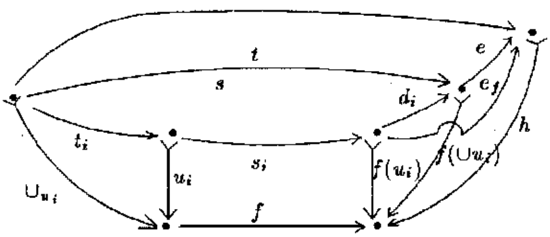
§2. Images and inverse images

Proposition 2.1 *For a quasiregular category, if $\mathrm{Im}(f) = \langle t \rangle$ and $u \leq t$, then $f^{-1}(u)$ exists.*

Proof f has a regular factorization $f = te$ with e regular epi and t monic. Since $u \leq t$, there is a monic s such that $ts = u$ and so that the diagram $(t, u; s, 1)$ is a pullback. Further, Axiom(QR) tells us there is a pullback $(e, s; a, b)$. Hence we have a pullback $(te, u; a, 1b)$ and so that $\langle a \rangle = f^{-1}(u)$.

Proposition 2.2 *For any category, if $\{u_i\}_{i \in T}$ is a family of subobjects $u_i's$ of f, and if $\cup_T u_i$, $f(\cup_T u_i)$ and $f(u_i)$ exists, then $\tilde{\cup}_T f(u_i) = f(\cup_T u_i)$.*

Proof The proof is a discussion on the following diagram.



Since $f(u_i)$ and $f(\cup u_i)$ exist, there exist commutative diagrams $(f, f(u_i); u_i, s_i)$ and $(f, f(\cup u_i) \cup u_i, s)$. By the definition of unions, we have a morphism t_i with $(\cup u_i)t_i = u_i$, so holds that $f(\cup u_i) \cdot (st_i) = (f \cdot (\cup u_i))t_i = fu_i$. So that, by the definition of images we have a unique d_i such that $f(\cup u_i)d_i = f(u_i)$ and hence $f(\cup u_i) \geq f(u_i)$.

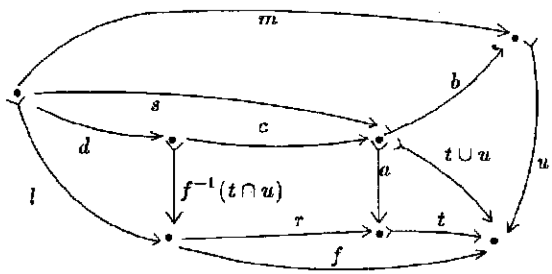
Suppose $f(u_i)$ is carried into h by 1, i.e., $he_i = f(u_i)$, then we have $h \cdot (e_i s_i) = f(u_i)s_i = fu_i$, so that u_i is carried into h by f, hence $\cup u_i$ is also carried into h by f and so that there is a morphism t such that $ht = f \cdot (\cup u_i)$. Because h is monic, the definition of images tells us that there is a unique e such that $he = f(\cup u_i)$, this means that $f(\cup u_i)$ is carried into h by 1 also, and hence $f(\cup u_i) = \tilde{\cup} f(u_i)$. □

Proposition 2.3 *For any category, if a_i is a subobject of a monic $f, i \in T$, and if $\cup a_i$ exists, then $\cup f(a_i) = f(\cup a_i)$.*

Proof In any category the image of a monic is the monic itself, so that $fa_j = f(a_j)$. Suppose $f(a_j)$ is carried into h by g, that is, there is a morphism s_j such that $hs_j = gf(a_j)$ and hence $hs_j = (gf)a_j$, so that a_j is carried into h by $gf. \cup a_i$ is carried into h by gf also, so that there is a morphism s such that $hs = (gf) \cup a_i$. We have $(gf) \cup a_i = g \cdot (f \cup a_i) = gf(\cup a_i)$, hence $gf(\cup a_i) = hs$ and so that $f(\cup a_i)$ is carried into h by g. On the other hand, by the definition of unions there is a morphism t_j such that $(\cup a_i)t_j = a_j$ for each j. To remark that t_j is monic, we have $f(a_j) = f((\cup a_j)t_j) = f \cdot ((\cup a_i)t_j) = f(\cup a_i)t_j$, so that $f(a_j) \leq f(\cup a_i)$. We have proved $f(\cup a_i) = \cup f(a_i)$. $\square$

Proposition 2.4 *For any category, if* $\mathrm{Im}(f) = \langle t \rangle$, *and if* $t \cap u$ *and* $f^{-1}(t \cap u)$ *exist, then* $f^{-1}(t \cap u) = f^{-1}(u)$.

Proof The proof can be completed by discussing the following diagram:



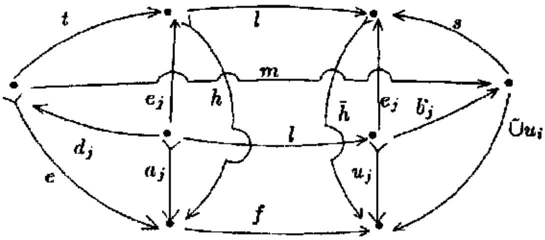
There is a morphism r such that $f = tr$, for t is the image of f. Since $t \cap u$ exists, there is a pullback $(t, u; a, b)$ with $t \cap u = ub = ta. f^{-1}(t \cap u)$ exists, i.e., there is a pullback $(f, t \cap u; f^{-1}(t \cap u), c)$. Hence we have $ff^{-1}(t \cap u) = (t \cap u)c = (ub)c = u(bc)$. If there are two morphisms l and m such that $fl = um$, then $t(rl) = fl = um$, so since $(t, u; a, b)$ is a pullback, we have a unique s such that $as = rl$ and $bs = m$, and hence $(ta)s = (tr)l$, so that $(t \cap u)s = fl$. Since $(f, t \cap u; f^{-1}(t \cap u), c)$ is a pullback, we have a unique d such that $cd = s$ and $f^{-1}(t \cap u)d = l$, and hence $(bc)d = b(cd) = bs = m$. To remark $f^{-1}(t \cap u)$ is monic, we have proved the diagram $(f, u; f^{-1}(t \cap u), bc)$ is a pullback. Hence $f^{-1}(t \cap u) = f^{-1}(u)$. $\square$

Proposition 2.5 *For any category, if f is monic, then $f^{-1}(\tilde{\cup} u_i) = \tilde{\cup} f^{-1}(u_i)$ when $\tilde{\cup} u_i$ exists and $u_i \leq f$.*

Proof We complete the proof by discussing the following diagram:

Since $u_j \leq f$, there is a morphism a_j such that $u_j = fa_j$. We may check that the diagram $(f, u_j; a_j, 1)$ is a pullback, so that $\langle a_j \rangle = f^{-1}(u_j)$. If a_j is carried into h by 1, i.e., there is a monic e_j such that $a_j = he_j$. Let $\bar{h} = fh$, then we have a pullback $(f, \bar{h}; h, 1)$ and $\bar{h}e_j = fhe_j = fa_j = u_j$, hence u_j is carried into $\bar{h}$ by 1, so that $\tilde{\cup} u_i$ is carried into $\bar{h}$ by 1 also, so we have a monic s such that $\bar{h}s = \tilde{\cup} u_i$. Since $u_j \leq f$, we have $\tilde{\cup} u_j \leq f$ and hence there is a monic e such that $fe = \tilde{\cup} u_i$, so that we have a pullback $(f, \tilde{\cup} u_i; e, m)$ with $m = 1$. Now we have $\bar{h} \cdot (sm) = (\tilde{\cup} u_i)m = fe$, so since $(f, \bar{h}; h, 1)$ is a pullback, we have a unique

t such that $ht = e$, and hence e is carried into h by 1. On the other hand, there is a monic b_j such that $u_j = (\bar{\cup} u_i) b_j$, so that $(\bar{\cup} u_i)(b_j 1) = f a_j$, so since $(f, \bar{\cup} u_i; e, m)$ is a pullback, we have a unique d_j such that $e d_j = a_j$ and hence $a_j \leq e$, so that $\langle e \rangle = \bar{\cup} a_i$. To observe that $(f, \bar{\cup} u_i; e, m)$ and $(f, u_j; a_j, 1)$ are pullbacks, we have proved $\langle e \rangle = f^{-1}(\bar{\cup} u_i) f^{-1}(u_i)$.



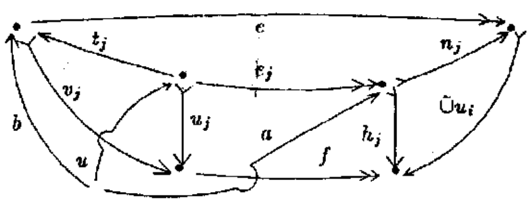
Since a morphism in a quasiregular category always has a regular factorization, this proposition shows why we mentioned merely epimorphisms as a condition in Axiom(U) when we hope to decide the property that f^{-1} preserves all existing unions in a quasiregular category (refer to proposions 2.7–2.8, and carefully see the proof of Proposition 2.8).

Proposition 2.6 *For any category, if f is monic, that $f^{-1}(\bar{\cup}(f \cap u_i)) = \bar{\cup} f^{-1}(u_i)$ when $f \cap u_i$ and $\bar{\cup}(f \cap u_i)$ exist.*

This is a corollary of Proposition 2.4 and Proposition 2.5.

Proposition 2.7 *If $\mathcal{A}$ is a (P)-weak regular category, then $\mathcal{A}$ satisfies Axiom(U)$\Leftrightarrow$ for every regular epi $f, \cup f^{-1}(h_i) = f^{-1}(\cup f_i)$ when $\cup h_i$ exists.*

Proof ($\Rightarrow$) We discuss the diagram

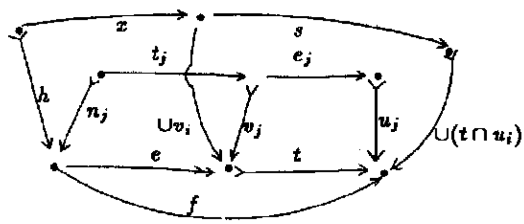


Since f is regular epi, from the second part of Axiom(QR) we know there are two pullbacks $(f, h_j; u_j, e_j)$ with e_j regular epi and $(f, \bar{\cup} h_i; u, e)$ with e regular epi, so that $\langle u \rangle = f^{-1}(\bar{\cup} h_i)$ and $\langle u_j \rangle = f^{-1}(h_j)$. Let n_j be a monic such that $(\bar{\cup} h_i) n_j = h_j$, then $(\bar{\cup} h_i) \cdot (n_j e_j) = f u_j$, so since $(f, \bar{\cup} h_i : u, e)$ is a pullback, there is a monic t_j such that $u t_j = u_j$ and $e t_j = n_j e_j$, hence the diagram $(e, n_j; t_j, e_j)$ commutes. If there are two morphisms a and b such that $n_j a = eb$, then we have $f \cdot (ub) = (fu) b = ((\bar{\cup} h_i) e) = (\bar{\cup} h_i) n_j a = h_j a$, so since $(f, h_j; u_j, e_j)$ is a pullback, there is a unique v_j such that $u_j v_j = ub$ and $e_j v_j = a$. Because we have shown $u t_j = u_j$, now we have $u_j v_j = u t_j v_j$, and hence $ub = u t_j v_j$. Since u is monic, we have $b = t_j v_j$. From both $e_j v_j = a$ and $b = t_j v_j$, and since t_j is monic, we know $(e, n_j; t_j, e_j)$ is a pullback, so that $\langle t_j \rangle = e^{-1}(n_j)$. On the other hand, by Lemma 1.5, we have $\bar{\cup} n_i = \langle 1 \rangle$, so since e is regular epi, Axiom(U) tells us $\bar{\cup} e^{-1}(n_i) = \bar{\cup} t_i = \langle 1 \rangle$. Moreover, by Lemma 1.3, we have $\cup t_i = \langle 1 \rangle$. Since u is monic, Proposition 2.3 shows $\cup u(t_i) = u(\cup t_i)$, hence it holds that $\cup u_i = \cup u(t_i) = u(\cup t_i) = u(1) = \langle u \rangle$, so that $\cup f^{-1}(h_i) = f^{-1}(\cup h_i)$.

($\Leftarrow$) Observe that $f^{-1}(1) = \langle 1 \rangle$, the proof is obvious. $\square$

Proposition 2.8 *If $\mathcal{A}$ is a (P)-quasiregular category, then $\mathcal{A}$ satisfies Axiom(U)$\Leftrightarrow$ for any morphism f with* $\mathrm{Im}(f) = \langle t \rangle$, *if* $u_i \cap t$ and $\cup(u_i \cap t)$ *exist then* $f^{-1}(\cup(u_i \cap t)) = \cup f^{-1}(u_i)$.

Proof We can prove "only if" by discussing the diagram



Let $f = te$ be a regular factorization of f. Since $t \cap u_j$ exists, we have a pullback $(t, u_j; v_j, e_j)$, and $\langle v_j \rangle = t^{-1}(u_j)$. To observe e is regular epi, Axiom(QR) tells us there is a pullback $(e, v_j; n_j, t_j)$, and $\langle n_j \rangle = e^{-1}(v_j)$, and hence $(f, u_j; n_j, e_j t_j)$ is a pullback and $\langle n_j \rangle = f^{-1}(u_j)$. Since $\cup(t \cap u_i)$ exists, by Corollary 2.6 we have $t^{-1}(\cup(t \cap u_i)) = \cup t^{-1}(u_i) = \cup v_i$, that is, there is a pullback $(t, \cup(t \cap u_i); \cup v_i, s)$. Furthermore, Axiom(QR) give us a pullback $(e, \cup v_i; h, x)$, so that $\langle h \rangle = f^{-1}(\cup(t \cap u_i)) = e^{-1}(\cup v_i)$. Thus, to observe Proposition 2.7, we have $f^{-1}(\cup(t \cap u_i)) = e^{-1}(\cup v_i) = \cup e^{-1}(v_i) = \cup n_i = \cup f^{-1}(u_i)$. $\square$

To review the above proofs carefully, we are convinced of that Axiom(P) has the ability to replace the finite-completeness when the relevant commutative diagrams exist, and many propositions, which people used to know to be true for the finitely complete categories, are true for some (P)-categories, for example, the finite-completeness in [10,I.3] can be replaced by Axiom(P) in many cases. It is interesting to compare the propositions in this paragraph with the relevant propositions in [10,I.3 and II].

If we regard AG as a special one of AG_{r_n}, then the finite-completeness of AG should be a degenerate form of Axiom(P): for one thing, in AG for any $\cdot \xrightarrow{f} \cdot \xleftarrow{g} \cdot$ there always exists a commutative diagram $(f, g; a, b)$, hence Axiom(P) says that we always have a pullback $(f, g; a', b')$, in additon, AG has a null object, so that it is finitely complete, for another, when $n > 2, AG_{r_n}$ satisfies Axiom(P), but it is not finitely complete, for may be $\mathrm{Im}(f) \cap \mathrm{Im}(g) = \varphi$ and so that there is no commutative diagram $(f, g; -, -)$. Thus, we can say $AG_{r_n}(n \geq 2)$ satisfies Axiom(P) and when $n = 2$ Axiom(P) degenerate into finite-completeness.

Since AG_{r_n} is an Ω-algebra, we have that Ω-alg is not finitely complete in general, but it satisfies Axiom(P).

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关于具有终对象的范畴的一些结论

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摘　要

本文对于具有终对象的范畴的一些结论进行了讨论，诸如在满足公理 (P) 和公理 (U) 的条件下的一些结果；另外，关于象与逆象的一些命题也进行了论证.