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On Compact Perturbations of m-Accretive Operators*

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Kartsatos and others proved surjectivity theorems for an m-accretive operator T perturbed by a compact mapping g under some geometric conditions involving variational inequalities and other assumptions (see [1, 2] and references there in). In this note, we introduce a simple condition (*) whenever g has bounded range $R(g)$. In particular, for some special cases, even condition (*) is not necessary. The results improve and extend the relevent results obtained before.

Theorem 1 *Let X be a Banach space, $T: D(T) \subset X \to 2^X$ be m-accretive and $g: D(T) \to X$ be a compact operator with bounded $R(g)$ (i.e., there is a constant $M > 0$ such that $\|y\| \le M$ for $y \in R(g)$). If there exist positive constants a and b such that*

$$|T(x)| \ge a\|x\| \qquad (*)$$

for $x \in D(T)$, $\|x\| \ge b$, where $|T(x)| = \inf\{\|y\|: y \in T(x)\}$, then $\overline{R(T+g)} = X$.

Sketch of Proof Define operator A_n by $A_n(u) = y - g(T + (1/n)I)^{-1}(u)$ for $y \in X$, then A_n has a fixed point u_n. Let $x_n = (T + (1/n)I)^{-1}(u_n)$ and select $v_n \in T(x_n)$ such that $v_n + (1/n)x_n = u_n$, we can prove that $\|v_n + g(x_n) - y\| = (1/n)\|x_n\| \to 0$ as $n \to \infty$.

Corollary 1 *Let X, T, g and M be as in theorem 1. If there is a constant $c > 0$ such that $(T + g)(\overline{B}(0; b + \frac{M+c}{a}) \cap D(T)$ is closed, then $\overline{B}(0; ab) \subset R(T+G)$.*

Corollary 2 *Let X, T be as in Theorem 1, $J_1 = (I+T)^{-1}$ be compact and $g: \overline{D}(T) \to X$ be a continuous operator with bounded $R(g)$. Then $R(T+g) = X$.*

Theorem 2 *Let X be a Banach space, $T: D(T) \subset X \to 2^X$ be m-accretive and $g: D(T) \to X$ be compact. If $D(T)$ is bounded, then $\overline{R(T+g)} = X$.*

Corollary 3 *Let X, T be as in Theorem 2, $J_1 = (I+T)^{-1}$ be compact and $g: \overline{D}(T) \to X$ be continuous. If $D(T)$ is bounded, then $R(T+g) = X$.*

References

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