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Remarks on the Mapping Theorems Involving Compact Perturbations of M - Accretive Operators in Banach Spaces*

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Abstract In the present paper, by virtue of new approch techniques, we obtain several mapping theorems involving compact perturbations of m —accretive operators. These results improve and extend the corresponding those obtained by Kartsatos, Zhu, and Kartsatos and Mabry.

Keywords compact perturbation, m —accretive operator, Yosida approximant.
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1. Introduction and Preliminaries

Let X be a real Banach space with norm $\|\cdot\|$ and a dual X^*. The normalized duality mapping $J: X \to 2^{X^*}$ is defined by

$$Jx = \{x^* \in X^*: \langle x, x^* \rangle = \|x\|^2 = \|x^*\|^2\},$$

where $\langle \cdot, \cdot \rangle$ denotes the generalized duality pairing. It is well known that if X^* is uniformly convex then J is single-valued and $J(tx) = tJx$, for all $t > 0, x \in X$, and J is uniformly continuous on bounded subsets of X. We denote the single- valued normalized duality mapping by j.

An operator T with domain $D(T)$ and range $R(T)$ in X is said to be compact, if it is continuous on $D(T)$ and maps bounded subset of $D(T)$ into relatively compact subset of X. T is completely continuos if it is continuous on $D(T)$ from the weak topology of X to the strong topology of X. An operator T is said to be accretive if for every $x, y \in D(T)$ there exists some $j(x-y) \in J(x-y)$ such that

$$\langle Tx - Ty, j(x-y) \rangle \geq 0. \tag{1}$$

For accretive operator, there is an equivalent definition (cf. Kato [4]). The operator T is accretive if and only if the inequality

$$\|x - y\| \leq \|x - y + s(Tx - Ty)\| \tag{2}$$

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holds, for every $x, y \in D(T)$ and $s > 0$. The operator T is said to be strongly accretive if there exists a positive constant k such that $T - kI$ is accretive, where $I: X \to X$ denotes the identity operator. An operator T is called m-accretive if T is accretive and $R(I + rT) = X$ for all $r > 0$. For an m-accretive operator T, the Yosida approximants J_n, T_n are defined by $J_n = (I + \frac{1}{n}T)^{-1}$, $T_n = TJ_n$, respectively. It is well known that the mappings J_n are nonexpansive and the mappings T_n are m-accretive and Lipschitzian continuous with Lipschitz constant $2n$. Moreover,

$$n(I - J_n) = TJ_n, \quad \|T_n x\| \leq \|Tx\|,$$

for all $x \in D(T)$, $n \geq 1$.

We denote by "$\to$" ("$\rightharpoonup$") strong (weak) convergence. We also denote by $B_b(0)$ the open ball with center at zero and radius $b > 0$. The symbols $\overline{D}$, $\partial\overline{D}$ denote the strong closure and the boundary of the set D, respectively. The letter R_+ denotes $(0, \infty)$.

Recently, several authors studied the solvability of the equations with the form

$$Tx + Cx = f,$$

where T is an m-accretive operator and C is compact.

In the present paper we continue the study of compact perturbatiobs of m-accretive operator. We show that the Yosida approximants TJ_n for a strongly m-accretive operator T are strongly m-accretive. The fact is used to prove our main results (Theorems 1-4). With our new approch, the assumption in [1, 2] that X^* is uniformly convex becomes unnecessary. Thus, our results are significant improvements of various results of Kartsatos [1, Theorem 6] and Zhu [2, Theorems 4-6].

Lemma 1.1 *Let $T: D(T) \subset X \to X$ be m-accretive and also be strongly accretive, then $TJ_n: X \to X$ are strongly m-accretive for all $n \geq 1$.*

Proof It is clear that TJ_n are m-accretive, for all $n \geq 1$. We only need to show that TJ_n are strongly accretive. Indeed, for every $x_1, x_2 \in X$, $J_n x_1, J_n x_2 \in D(T)$, since T is strongly accretive, there is some constant $k > 0$ such that

$$\langle TJ_n x_1 - TJ_n x_2, j\rangle \geq k\|J_n x_1 - J_n x_2\|^2,$$

for some $j \in J(J_n x_1 - J_n x_2)$.

By the equality $n(I - J_n) = TJ_n$, we have

$$n\langle x_1 - x_2 - (J_n x_1 - J_n x_2), j\rangle \geq k\|J_n x_1 - J - n x_2\|^2, \tag{3}$$

which leads to

$$n(\langle x_1 - x_2, j\rangle - \|J_n x_1 - J_n x_2\|^2) \geq k\|J_n x_1 - J_n x_2\|^2. \tag{4}$$

It follows from (4) that

$$\|J_n x_1 - J_n x_2\| \leq (1 + \frac{k}{n})^{-1}\|x_1 - x_2\|. \tag{5}$$

At this point, choosing $j_1 \in J(x_1 - x_2)$, we have

$$\langle TJ_n x_1 - TJ_n x_2, j_1 \rangle = n\langle x_1 - x_2 - (J_n x_1 - J_n x_2), j_1 \rangle$$
$$\geq n(\|x_1 - x_2\|^2 - (1 + \frac{k}{n})^{-1}\|x_1 - x_2\|^2)$$
$$= \frac{k}{1 + \frac{k}{n}}\|x_1 - x_2\|^2$$
$$\geq \frac{k}{1 + k}\|x_1 - x_2\|^2,$$

for all $n \geq 1$, completing the proof of Lemma 1.1. □

Lemma 1.2 *Let $T, C: X \to X$ be bounded strongly m-accretive and compact, respectively. Assume that there exist constants $b, c > 0$ such that*

$$< Cx, j > \geq -c\|x\|,$$

for every $\|x\| \geq b$, for every $j \in Jx$, then the equation

$$Tx + Cx = f$$

has at least one solution for any $f \in X$.

Proof Let f be given. Since T is strongly accretive we know that for every $x \in X$, there exists $j \in Jx$ such that

$$\langle Tx - T0, j \rangle \geq k\|x\|^2, \tag{6}$$

where k is a strongly accretive constant for T. By taking $r = \max\{b, \frac{c + \|T0\| + \|f\|}{k}\}$, we have

$$\langle Tx + Cx - f, j \rangle \geq \langle Tx - T0, j \rangle + \langle Cx, j \rangle + \langle T0, j \rangle - \|f\|\|x\|$$
$$\geq k\|x\|^2 - (\|T0\| + c + \|f\|)\|x\|$$
$$\geq 0,$$

for every $x \in \partial B_r(0)$, for some $j \in Jx$. By Chen [3, Theorem 5], we see that

$$f \in (T + C)(\overline{B_r(0)}),$$

which shows the equation $Tx + Cx = f$ has at least one solution for any $f \in X$. The proof is complete. □

Lemma 1.3 *Let $T: D(T) \subset X \to X$ be m-accretive, then for any sequence $\{x_n\} \subset D(T)$ such that $x_n \to x \in X$, $Tx_n \to y \in X$ as $n \to \infty$, we have $x \in D(T)$ and $y = Tx$.*

Proof For any $u \in D(T)$, by the equivalent definition of accretive operator T, we have

$$\|u - x_n\| \leq \|u - x_n + (Tu - Tx_n)\|. \tag{7}$$

By taking limit on the both sides of (7), we obtain

$$\|u - x\| \leq \|u - x + (Tu - y)\|. \tag{8}$$

Letting $u = (I + T)^{-1}(x + y)$, we get that $u + Tu = x + y$, i.e., $u - x = y - Tu$. Thus, $\|u - x\| \leq \|y - Tu + (Tu - y)\| = 0$. So, we have $x = u \in D(T)$ and $y = Tu = Tx$, completing the proof of Lemma 1.3. □

Lemma 1.4 *Let $T: D(T) \subset X \to X$ be an m-accretive operator and k be a positive constant, then $\|J_n^k x - J_n x\| \to 0$ as $n \to \infty$, where $J_n^k = [I + \frac{1}{n}(T + kI)]^{-1}$ and $J_n = (I + \frac{1}{n}T)^{-1}$.*

Proof Set $u_n^k = J_n^k x$ and $u_n = J_n x$, then

$$u_n^k + \frac{1}{n}Tu_n^k + \frac{k}{n}u_n^k = x, \quad u_n + \frac{1}{n}Tu_n = x. \tag{9}$$

It follows from (9) that

$$\frac{k}{n}u_n^k = \frac{1}{n}(Tu_n - Tu_n^k) + u_n - u_n^k. \tag{10}$$

By the definition of accretive operator T, we have

$$\frac{k}{n}\|u_n^k\| \geq \|u_n - u_n^k + \frac{1}{n}(Tu_n - Tu_n^k)\| \geq \|u_n - u_n^k\|. \tag{11}$$

We now pick a fixed element $x_0 \in D(T)$, then $J_n^k x_0 \to x_0$ as $n \to \infty$. Since $J_n^k: X \to D(T)$ is non-expansive, we have

$$\|u_n^k\| = \|J_n^k x\| \leq \|J_n^k x - J_n^k x_0\| + \|J_n^k x_0\| \leq \|x - x_0\| + M, \tag{12}$$

where M is some positive constant such that $\|J_n^k x_0\| \leq M$ for all large n. Therefore we have $\frac{k}{n}u_n^k \to 0$ as $n \to \infty$, and hence $\|u_n^k - u_n\| \to 0$ as $n \to \infty$. The proof of Lemma 1.4 is complete. □

Lemma 1.5 *Let $T: D(T) \subset X \to X$, $C: D(C) \subset X \to X$ be strongly m-accretive and compact, respectively.*

(1) *if $D(C) = \overline{D(T)}$ and the equation*

$$TJ_n x + CJ_n x = f \tag{13}$$

has a solution $x_n \in \overline{B_b(0)}$, for every $n = 1, 2, \cdots$, then the equation

$$Tx + Cx = f \tag{14}$$

has a solution $x \in \overline{B_b(0)} \cap D(T)$.

(2) *if $D(C) = X$ or $\overline{B_b(0)}$ and the equation*

$$TJ_n x + Cx = f \tag{15}$$

has a solution $x_n \in \overline{B_b(0)}$, for every $n = 1, 2, \cdots$, then the equation $Tx + Cx = f$ has a solution $x \in \overline{B_b(0)} \cap D(T)$.

Proof (1) We pick a fixed $x_0 \in D(T)$. Since J_n are nonexpansive and $J_n x_0 \to x_0$ as $n \to \infty$, we assert that there exists a fixed constant $M > 0$ such that

$$\|J_n x_n\| \leq \|J_n x_0\| + \|x_n - x_0\|,$$

for all large n. Letting $u_n = J_n x_n$, we may assume that $Cu_n \to y$ as $n \to \infty$. Thus $Tu_n \to f - y$ as $n \to \infty$. Hence $\{Tu_n\}$ is a Cauchy sequence. Since T is strongly accretive, we have

$$\|Tu_n - Tu_m\| \geq k\|u_n - u_m\|,$$

for all large n,m. It follows that $\{u_n\}$ is Cauchy. Assume $u_n \to u \in \overline{D(T)}$, then $Cu_n \to Cu$ and $Tu_n \to f - Cu$ as $n \to \infty$. By Lemma 1.3, we see that $u \in D(T)$ and $Tu + Cu = f$.

Noting that $\|J_n x_n - x_n\| = \frac{1}{n}\|TJ_n x_n\|$, we know that

$$\|x_n - u\| = \|x_n - u_n + u_n - u\| \leq \frac{1}{n}\|Tu_n\| + \|u_n - u\| \to 0$$

as $n \to \infty$. Hence $x_n \to u$ as $n \to \infty$. Since $x_n \in \overline{B_b(0)}$, we see that $u \in \overline{B_b(0)}$. The proof of (2) is similar to the proof of (1) above, so we omit it. We complete the proof of Lemma 1.5. □

2 Main Results

Now we prove the main results of this paper.

Theorem 2.1 *Let X be a real Banach space which is uniformly convex and let $T: D(T) \subset X \to X$ be m-accretive, $C: \overline{B_b(0)} \to X$ completely continuous. Assume that $0 \in D(T)$ and there exists constant $r > 0$ such that for all $x \in \partial B_b(0)$ and $j \in Jx$,*

$$\langle Cx, j\rangle \geq (\|T0\| + r)b. \tag{16}$$

Then $\overline{B_r(0)} \subset (T+C)(\overline{B_b(0)} \cap D(T))$.

Proof We consider the following approximating problem:

$$TJ_n^k x + Cx + kJ_n^k x = f, \tag{17}$$

where $f \in \overline{B_r(0)}$ and $k > 0$ are given. For every $x \in \partial B_b(0)$, there exists some $j \in Jx$ such that

$$\begin{aligned}\langle (T+kI)J_n^k x + Cx - f, j\rangle &= \langle (T+kI)J_n^k x - (T+kI)J_n^k 0, j\rangle \\ &\quad + \langle (T+kI)J_n^k 0, j\rangle + \langle Cx - f, j\rangle \\ &\geq \frac{k}{1+k}\|x\|^2 + (\|T0\| + r)b - (\|T0\| + r)b \\ &= \frac{k}{1+k}b^2 > 0.\end{aligned}$$

Here we have used the fact $\langle (T+kI)J_n^k x - (T+kI)J_n^k 0, j\rangle \geq \frac{k}{1+k}\|x\|^2$.

By Chen [3, Theorem 5], we assert that the equation

$$(T + kI)J_n^k x + Cx = f$$

has a solution $x_n \in \overline{B_b(0)}$ for every $n = 1, 2, \cdots$. By Lemma 1.5 we know that the equation

$$(T + kI)x + Cx = f$$

has a solution $x \in \overline{B_b(0)} \cap D(T)$.

We now choosing $k_n > 0$ such that $k_n \to 0$ as $n \to \infty$, then there exists $x_n \in \overline{B_b(0)} \cap D(T)$ such that

$$Tx_n + Cx_n + k_n x_n = f.$$

Since X is reflexive, we may assume that $x_n \rightharpoonup x \in \overline{B_b(0)}$ as $n \to \infty$. By Kartsatos [1, Lemma 1] we see that $Tx + Cx = f$, or $f \in (T + C)(\overline{B_b(0)} \cap D(T))$, completing the proof of Theorem 2.1. □

Remark 1 *Theorem 2.1 shows that Theorem 6 of Kartsatos [1] holds true without assumption that X^* is uniformly convex. Thus our result improves the corresponding that of Kartsatos [1].*

Theorem 2.2 *Let $T: D(T) \subset X \to X$, $C: X \to X$ be m-accretive and compact, respectively. Assume that $0 \in D(T)$ and there exist constants $r > 0$, $b > 2(\|T0\| + r)$ and $a > 3$ such that*

(*i*) $\|Tx + Cx\| \geq a(\|T0\| + r)$, $x \in D(T)$, $\|x\| \geq b$;

(*ii*) $\langle Cx, j \rangle \geq -(\|T0\| + r)\|x\|$, $\|x\| \geq b$, $j \in Jx$.

Then $\overline{B_r(0)} \subset \overline{R(T + C)}$. If, moreover, C is completely continuous and X is uniformly convex, then $\overline{B_r(0)} \subset R)T + C)$.

Proof We first consider the equations

$$TJ_n^k x + Cx + kJ_n^k x = f, \quad n = 1, 2, \cdots, \tag{18}$$

for a fixed $f \in \overline{B_r(0)}$, where $k = \dfrac{2(\|T0\| + r)}{b - 2(\|T0\| + r)}$.

Let $x \in \partial B_b(0)$, $j \in Jx$, then

$$\begin{aligned}\langle (T + kI)J_n^k x + Cx - f, j \rangle &= \langle (T + kI)J_n^k x - (T + kI)J_n^k 0, j \rangle \\ &\quad + \langle (T + kI)J_n^k 0, j \rangle + \langle Cx - f, j \rangle \\ &\geq \frac{k}{1 + k} b^2 - 2(\|T0\| + r)b \geq 0.\end{aligned}$$

Here we have used Lemma 1.1 and the fact $\|(T + kI)J_n^k 0\| \leq \|T0\|$. By Chen [3, Theorem 5], we know that the equation (18) has a solution $x_n \in \overline{B_b(0)}$ for each $n = 1, 2, \cdots$. Then, by Lemma 1.5, the equation

$$Tx + Cx + kx = f \tag{19}$$

has a solution $x \in \overline{B_b(0)} \cap D(T)$.

Let $k_m = \dfrac{2(\|T0\| + r)}{m - 2(\|T)\| + r)}$ with $m > 2(\|T0\| + r)$, then the equation

$$Tx + Cx + k_m x = f \tag{20}$$

has a solution $x_m \in \overline{B_m(0)} \cap D(T)$, for $m > 2(\|T0\| + r)$. As in the proof of Zhu [1, Theorem 4], we can show that $\|x_m\| < b$, for $m > 2(\|T0\| + r)$. Therefore, we have $Tx_m + Cx_m \to f$ as $m \to \infty$, i.e., $f \in \overline{(T + C)(B_b(0) \cap D(T))}$. The second part of the conclusions is obvious, completing the proof of Theorem 2.2. □

Theorem 2.3 *Let $T: D(T) \subset X \to X$, $C: \overline{D(T)} \to X$ be m-accretive and compact, respectively. Assume that $0 \in D(T)$ and there exist constants $r > 0$, $b > 2(\|T0\| + r)$ and $a > 3$ such that*

(*i*) $\|Tx + Cx\| \geq a(\|T0\| + r)$, $x \in D(T)$, $\|x\| \geq b$;

(*ii*) $\langle CJ_n x, j\rangle \geq -(\|T0\| + r)\|x\|$, $x \in X$, $\|x\| \geq b$, *for all large n and $j \in Jx$. Then the conclusions of Theorem 2.2 hold.*

Proof We first consider the equations

$$TJ_n^k x + CJ_n x + kJ_n^k x = f, n = 1, 2, \cdots, \tag{21}$$

where $f \in \overline{B_r(0)}$, $k = \dfrac{2(\|T0\| + r)}{b - 2(\|T0\| + r)}$.

Let $x \in \partial B_b(0)$, $j \in Jx$, then

$$\begin{aligned}\langle (T + kI)J_n^k x + CJ_n x - f, j\rangle &= \langle (T + kI)J_n^k x - (T + kI)J_n^k 0, j\rangle \\ &\quad + \langle (T + kI)J_n^k 0, j\rangle + \langle CJ_n x, j\rangle - \langle f, j\rangle \\ &\geq \frac{k}{1+k}\|x\|^2 - 2(\|T0\| + r)\|x\| \geq 0.\end{aligned}$$

By Chen [3, Theorem 5], the equation (21) has a solution $x_n \in \overline{B_b(0)}$ for all large n. Set $u_n^k = J_n^k x_n$, $u_n = J_n x_n$, then we have

$$Tu_n^k + Cu_n + ku_n^k = f. \tag{22}$$

Since $\{u_n\}$ is bounded, without loss of generality, we may assume that $Cu_n \to y$ as $n \to \infty$. So, $Tu_n^k + ku_n^k \to f - y$ as $n \to \infty$. Since $(T + kI)$ is strongly accretive, we see that $\{u_n^k\}$ must be Cauchy. Thus, $u_n^k \to u$ as $n \to \infty$. By Lemma 1.4 we know that $\|u_n^k - u_n\| \to 0$ as $n \to \infty$. Consequently, $u_n \to u$ and $Cu_n \to Cu$ as $n \to \infty$. By Lemma 1.3 we have $Tu + Cu + ku = f$. Since $0 \in D(T)$, we have $J_n 0 \to 0$ as $n \to \infty$. Noting that $\|u_n\| = \|J_n x_n\| \leq \|J_n 0\| + \|x_n\|$, we know that $\|u\| \leq b$, i.e., $u \in \overline{B_b(0)} \cap D(T)$.

Let $k_m = \dfrac{2(\|T0\| + r)}{m - 2(\|T0\| + r)}$ with $m > 2(\|T0\| + r)$, then, as in the proof above, the equation

$$Tx + Cx + k_m x = f, \tag{23}$$

has a solution $x_m \in \overline{B_m(0)} \cap D(T)$, for every $m > 2(\|T0\| + r)$. Moreover, we can show that $\|x_m\| < b$, for all $m > 2(\|T0\| + r)$, similar to the proof of Theorem 2.2, it is therefore omitted. We complete the proof of Theorem 2.3. □

Remark 2 *Theorems 2, 3 show that Theorems 4, 4' hold true without assumption that X^* is uniformly convex. Thus our results are significant improvements of Zhu [2, Theorems 4, 4']. Also, we can prove that Theorems 5, 6 of Zhu [2] hold true without assumption that X^* is uniformly convex.*

Theorem 2.4 *Let X be reflexive and let $A: D(A) \subset X \to X$ be a strongly m-accretive operator with $0 = A0$, $T: X \to X$ a linear compact operator and $C: \overline{D(A)} \to X$ a completely continuous operator. Assume that there exists a completely continuous function $g: \overline{B_1(0)} \to R_+$ such that $g(u) = 0$ implies $u = 0$ and*

$$\langle Cu, j\rangle \geq g\left(\frac{u}{\|u\|}\right)\|u\|^{p+1},$$

for fixed $p > 1$, $u \in \overline{D(A)} \setminus \{0\}$, $j \in Ju$.

Then $Au - \lambda Tu + Cu = f$ is solvable for all $\lambda \in R_+$, $f \in X$.

Proof For the sake of simplicity, we prove Theorem 2.4 only for $\lambda = 1$. We first consider approximating problems:

$$AJ_n x - TJ_n x + CJ_n x + \frac{1}{n}x = f, \quad n = 1, 2, \cdots, \tag{24}$$

for fixed $f \in X$, where $J_n = (A + \frac{1}{n}I)^{-1}$. Since $A: D(A) \subset X \to X$ is strongly m-accretive, we know that there exists some positive constant b_1 such that $f \in (\frac{1}{n}I + AJ_n)(B_{b_1}(0))$. By Chen [3, Theorem 3.3], we have

$$\deg\left(\frac{1}{n}I + AJ_n, B_{b_1}(0), f\right) = 1$$

for all n. We shall prove that there exists a positive constant b_2 such that

$$\frac{1}{n}x + AJ_n x - t(TJ_n x - CJ_n x) \neq f$$

for all $t \in [0, 1]$, $x \in \partial B_{b_2}$. Indeed, if it is not the case, then there exist some sequence $\{t_m\} \subset [0, 1]$, and $\{x_m\} \subset X$ with $\|x_m\| \to \infty$ such that

$$\frac{1}{n}x_m + AJ_n x_m - t_m(TJ_n x_m - CJ_n x_m) = f. \tag{25}$$

Set $u_m = J_n x_m$, then $u_m \in D(A) \setminus \{0\}$. From (25) we obtain

$$\left(1 + \frac{1}{n^2}\right)Au_m - t_m Tu_m + t_m Cu_m + \frac{1}{n}u_m = f. \tag{26}$$

We assert that there exists a constant $k > 0$ such that

$$\inf_{m \geq 1} g\left(\frac{u_m}{\|u_m\|}\right) \geq k.$$

Assume the contrary and let $v_m = \frac{u_m}{\|u_m\|}$ have a subsequence, denoted again by $\{v_m\}$, such that $g(v_m) \to 0$ as $m \to \infty$. Then we have $v_m \rightharpoonup v$ as $m \to \infty$, since X is reflexive and $\{v_m\}$ is bounded and hence $g(v_m) \to g(v)$ as $m \to \infty$. Thus, $g(v) = 0$. By our assumption on g, we have $v = 0$, therefore $v_m \rightharpoonup 0$ as $m \to \infty$.

Now choosing some $j \in Ju_m$, we have

$$c\|u_m\|^2 \leq \langle Au_m, j\rangle \leq t_m\langle Tu_m, j\rangle + \langle f, j\rangle, \tag{27}$$

where c is a strongly accretive constant for operator A.

Divided by $\|u_m\|^2$ on the both sides of inequality (27), it gives to

$$c \leq t_m\langle Tv_m, \frac{j}{\|u_m\|}\rangle + \frac{1}{\|u_m\|}\langle f, \frac{j}{\|u_m\|}\rangle. \tag{28}$$

Observing that $Tv_m \to T0 = 0$ and $\frac{1}{\|u_m\|} \to 0$ as $m \to \infty$, we have $c \leq 0$, a contradiction. It follows that, for some constant $k > 0$, $\langle Cu_m, j\rangle \geq k\|u_m\|^{p+1}$, for every $m = 1, 2, \cdots$. Taking this fact into consideration in (26), we obtain that

$$0 = (1 + \frac{1}{n^2})\langle Au_m, j\rangle - t_m\langle Tu_m, j\rangle + t_m\langle Cu_m, j\rangle + \frac{1}{n}\|u_m\|^2 - \langle f, j\rangle$$

$$\geq c\|u_m\|^2 - t_m\|T\|\|u_m\|^2 + kt_m\|u_m\|^{p+1} - \|f\|\|u_m\|.$$

Now we consider two possible cases

Case 1 $t_m \to 0$. *In the case, we can choose* m *so large that* $c - t_m\|T\| \geq \frac{c}{2}$, *thus we get a contradiction with* $0 \geq \frac{c}{2}\|u_m\|^2 - \|f\|\|u_m\|$.

Case 2 $t_m \to t$. *In the case, we can choose* m *so large that* $t_m \geq \frac{t}{2}$, *thus we also get a contradiction with* $0 \geq \frac{t}{2}(k\|u_m\|^{p+1} - \|T\|\|u_m\|^2)$.

Consequently, there exists a positive constant b_2 such that

$$\frac{1}{n}x + AJ_nx - t(TJ_nx - CJ_nx) \neq f,$$

for all $t \in [0, 1]$ and $x \in \partial B_{b_2}(0)$.

Choosing $r = \max\{b_1, b_2\}$, we have

$$deg(AJ_n - TJ_n + CJ_n + \frac{1}{n}I, B_r(0), f) = deg(\frac{1}{n}I + AJ_n, B_r(0), f) = 1, \tag{29}$$

and hence $AJ_nx + CJ_nx - TJ_nx + \frac{1}{n}x = f$ has at least one solution $x_n \in B_r(0)$ for each $n = 1, 2, \cdots$.

Letting $u_n = J_nx_n$, then we obtain

$$(1 + \frac{1}{n^2})Au_n - Tu_n + Cu_n + \frac{1}{n}u_n = f,$$

by Lemma 1.5, we see that there exists some $u \in \overline{B_r(0)} \cap D(A)$ such that

$$Au - Tu + Cu = f,$$

completing the proof of Theorem 2.4. □

Remark 3 *If* X *is a real separable Hilbert space, then we obtain Theorem 3 of Kartsatos and Mabry [7]. Hence, our Theorem 2.4 extends the result of Kartsatos and Mabry [7].*

References

[1] A. G. Kartsatos, *On the solvability of abstract operator equations involving compact perturbations of m - accretive operators*, Nonl. Anal., **11**(1987), 997-1004.

[2] L. Zhu, *Mapping theorems for compact perturbations of m - accretive operators*, J. Math. Anal. Appl., **174**(1993), 413-427.

[3] Y. Z. Chen, *The generalized degree for compact perturbations of m - accretive operators and applications*, Nonl. Anal., **4**(1989), 393-403.

[4] T. Kato, *Nonlinear semigroups and evolution equations*, J. Math. Soc. Japan, **18/19**(1967), 508-520.

[5] V. Barbu, *Nonlinear Semigroups and Differential Equations in Banach Spaces*, Noordhoff Int. Publ. Leyden, The Netherlands, (1976)

[6] K. Deimling, *Nonlinear Functional Analysis*, Springer-Verlag, New York/Berlin, (1985)

[7] A. G. Kartsatos and R. D. Mabry, *On the solvability in Hilbert space of certain nonliear operator equations depending on parameters*, J. Math. Anal. Appl., **120**(1986), 670-678.

Banach 空间中含 m - 增生算子紧扰动的映象定理的若干注记

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摘 要

借助于某些新的逼近技巧得到了几个含 m - 增生算子紧扰动的映象定理. 这些结果改进并扩展了由 Kartsatos, Zhu **和** Kartsatos and Mabry **所建立的相应结果.**