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The Fine Continuity for Complex Generalized Weights*

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Abstract In this paper the notions of fine complex weights and the Choquet type for complex weights are introduced. And we discuss the relations between quasi-continuity and fine continuity quasi-everywhere.

Keywords fine complex weight, Choquet type, Fine-(Quasi-)continuous

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1 The Notion of Complex Weights

Let Y be a set, Φ: $\Phi(Y)$ a convex cone composed of non-negative numberical functions on Y satisfying the upper directed axiom. Let $\mathbf{F} := \mathbf{F}(Y)$ be an extended complex-valued function family. And $\mathbf{R}_e^{\pm}\mathbf{F}$, $\mathbf{I}_m^{\pm}\mathbf{F}$ and $|\mathbf{F}|$ are all included in Φ.

Let $\langle X, T\rangle$ be a Hausdorff topology space. Assume there is another topology $\widetilde{T}$ finer than T in X. Here and hereafer, we will use "fine" before the notions connected with $\widetilde{T}$ in order to differentiate from T. And denote by $\overline{A}$ (resp. $\widetilde{A}$) the closure of a subset $A \subset X$ with respect to T (resp. $\widetilde{T}$). A set map $W: 2^X \to \mathbf{F}$ is called a complex p-weight on X if $W(\emptyset) = 0$. A complex p-weight with the open-major property is called a complex (generalized) weight, with respect to which we use the notions and notations in [4, 5].

2 Quasi-Continuity and Fine Continuity Quasi-Everywhere

A complex weight W is said to be fine if for any $A \in 2^X$, $W(\widetilde{A}) = W(A)$.

Obviously $W_M(A) := |W(A)|$ is fine if W is; and W is fine if and only if each W_j is, $1 \leq j \leq 4$, where $W_1 = W_R^+$, $W_2 = W_R^-$, $W_3 = W_I^+$, $W_4 = W_I^-$; W_R and W_I are real and imaginary parts of W respectively.

Theorem 2.1 *Let X and X' are both T_2 spaces, W a fine complex weight sub-strongly right continuous. Then we have*

(*i*) *If the map $f: X \to X'$ is W-quasi-continuous, then f is fine continuous W-quasi-everywhere.*

(*ii*) *If f is W-quasi-upper (resp. lower)-semicontinuous real function on X, then f is fine upper (resp. lower)-semicontinuous W-quasi-everywhere.*

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Corollary 2.1 *Let W be a fine complex weight strongly right continuous. If A is a quasi-closed-set, then $W(\tilde{A} \setminus A) = 0$; if A is a quasi-open-set, then $W(A \setminus A_0) = 0$, where A_0 is fine inner of A.*

Let W be a complex p-weight. If any fine open set is W-quasi-(resp. strong W-quasi-) open-set, then we say W is of generalized Choquet (resp. strong Choquet) type.

Theorem 2.2 *Let X and X' be both T_2 space. Suppose the complex p-weight W is totally countably para-subadditive, and is of generalized strong Choquet type. Then*

(i) if the map $f: X \to X'$, X' with countable base, is fine continuous W-quasi-everywhere, then f is W-quasi-continuous;

(ii) if f is fine upper (resp. lower)-semicontinuous real function W-quasi-everywhere, then f is stromglt W-quasi-upper (resp. lower)-semicontinuous.

Corollary 2.2 *Let W be a complex weight strongly right continuous. Then W is of generalized Choquet (resp. strong Choquet) type if and only if, for any characteristic function χ_A of $A \in 2^X$, that χ_A is fine upper-semicontinuous implies that χ_A is W-quasi-upper-semicontinuous.*

From Theorem 2.1 and Theorem 2.2, we conclude:

Theorem 2.3 *Let X and X' be both T_2 space. If the complex weight W is fine, totally countably para-subadditive, sub-strongly right continuous and is of generalized strong Choquet type, then*

(i) for any the map $f: X \to X'$ (X' with countable base), f is W-quasi-continuous if and only if f is fine continuous W-quasi-everywhere;

(ii) for any real function f, f is W-quasi-upper (resp. lower)-semicontinuous if and only if f is fine upper (resp. lower)-semicontinuous W-quasi-everywhere.

3 Choquet Property

Let W be a complex p-weight. If for any $e \in 2^X$, there exists a real (resp. real and bounded) function $u \in \Phi$ such that for any number $\epsilon > 0$, there is an open set ω satisfying $\mathbf{C}\tilde{e} \subset \omega$ and $W_j(\omega \cap e) \le \epsilon u$ $(1 \le j \le 4)$, then we say W is of quasi-Choquet (resp. strong quasi-Choquet) type, where $\mathbf{C}\tilde{e}$ is the fine outer part of $\tilde{e}$.

Theorem 3.1 *Let W be a complex p-weight. We have*

(i) if W is of quasi-Choquet (resp. strong quasi-Choquet) type, then W is of (strong) generalized Choquet (resp. strong Choquet) type;

(ii) if W is sub-strongly (resp. strongly) para-right continuous and of generalized Choquet (resp. strong Choquet) type, then W is of quasi-Choquet (resp. strong quasi-Choquet) type.

Proof (i) Let A be a fine open set, then the complementry $e := \mathbf{C}A$ of A is fine closed. By the hypothesis, there exists a real (resp. real and bounded) $u \in \Phi$ such that for any number $\epsilon > 0$, there is an open set ω satisfying $A = \mathbf{C}\tilde{e} \subset \omega$ and $W_j(\omega \cap e) \le \epsilon u$. Evidently, $\omega \setminus A = \omega \cap \tilde{e} = \omega \cap e$. We have $W_j(\omega \setminus A) = W_j(\omega \cap e) \le \epsilon u$ $(1 \le j \le 4)$, which implies A is W-quasi-(resp. strong W-quasi-)open.

(ii) For any $e\in 2^X$, the fine outer part $\mathbf{C}\tilde{e}$ of $\tilde{e}$ is fine open, then $\mathbf{C}\tilde{e}$ is W-quasi-(resp. strong W-quasi-)open. So there exists a real (resp. real and bounded) $u\in\Phi$ such that for any number $\epsilon>0$, there is an open set $\omega\supset\mathbf{C}\tilde{e}$ satisfying $W_j(\omega\setminus\mathbf{C}\tilde{e})\leq\epsilon u/K$, where K is the positive number with respect to para-right continuity of W. Therefore there exists a bounded (resp. real and bounded) $u_1\in\Phi$ such that there is an open set G satisfying $\omega\setminus\mathbf{C}\tilde{e}\subset G$ and $W_j(G)\leq KW_j(\omega\setminus\mathbf{C}\tilde{e})+\epsilon u_1$. Consequently we have $W_j(G)\leq\epsilon u_0$, where $u_0:=u+u_1\in\Phi$. Obviously $\omega\cap e\subset\omega\cap\tilde{e}=\omega\setminus\mathbf{C}\tilde{e}$, which implies $W_j(\omega\cap e)\leq W_j(G)\leq\epsilon u_0\,(1\leq j\leq 4)$.

Theroem 3.2 *Let W be a complex weight.*

(*i*) *Let W is strongly right continuous and totally and countably sub-additive. Then the generalized strong Choquet property is equivalent to: for any real f, f is fine upper-semicontinuous implies f is strongly quasi-upper-semicontinuous.*

(*ii*) *Let W is fine, strongly right continuous and totally countably subadditive. Then the generalized strong Choquet property is equivalent to the identity of "fine upper-semicontinuous W-quasi-everywhere" and "strong quasi-upper-semicontinuous" for any real function f.*

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复广义权的细连续性

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摘　要

本文引入细复广义权和Choquet型复广义权的概念.讨论了某些与复广义权相关的函数的拟连续性与细拟处处连续的关系.