弹性振动点观测反馈控制古典解的存在性*

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摘　要: 本文运用有界线性算子的积分半群理论证明弹性振动点观测反馈控制方程古典解的存在性

关键词: 弹性振动反馈控制, 积分半群, 古典解
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细长飞行器在飞行控制中, 对角度, 角速度及加速度的量测, 实际上都是点观测 以往的处理方法是用包含观测点的小区域上的平均值来代替那一点的观测值, 以便用通常的偏微分方程的L^p 理论来处理 严格来说, 对这类观测量的描述要用到广义函数 本文运用有界线性算子的积分半群理论来直接处理这类问题, 证明了它的古典解的存在性, 这在理论和实际上都是很有意义的

我们用两端固定的长为 l 的均匀梁来近似描述细长飞行器, 在其上设置观测点, 以在这些点(x_1, x_2, …, x_n) 所测得的角速度作为反馈控制律的弹性振动闭环系统可用如下偏微分方程来描述[1, 2]:

$$\begin{aligned}
&\frac{\partial^2 y(x,t)}{\partial t^2} + b(x)\sum_{i=1}^{n} k_i \frac{\partial^2 y(x,t)}{\partial x\,\partial t}\Big|_{x=x_i} + \frac{\partial^4 y(x,t)}{\partial x^4} = 0, \qquad 0<x<l,\\
&y(x,t)\Big|_{x=0,l} = y'(x,t)\Big|_{x=0,l} = 0, \qquad\qquad (1.1)\\
&y(x,0) = y_0(x), \qquad y'(x,0) = y_1(x).
\end{aligned}$$

其中 $b(x)$ 为外力分布, k_i为量测量的放大系数 为讨论方便, 不妨设 $k_i = 1, i = 1, 2, \cdots, n$.

设 $C[0, l]$ 为 $[0, l]$ 上的连续函数在通常的范数 $\|\cdot\|$下所组成的Banach 空间, $C^1[0, l]$ 为 $[0, l]$上的一阶连续可微函数的全体在范数 $\|y\|_1 = \|y\| + \|y'\|$ 下所构成的Banach 空间 在 $C^1[0, l]$ 上定义线性算子: $(\mathbf{A}y)(x) = \dfrac{d^4 y(x)}{dx^4}$,

$$D(\mathbf{A}) = \{y \in C^1[0, l] \,|\, y', y'', y''', y^{(4)} \in C^1[0, l],$$

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$$y(x)\big|_{x=0,l}=y''(x)\big|_{x=0,l}=0\};$$

$$(\mathbf{B}y)(x)=b(x)\sum_{i=1}^{n}y'(x_i),\quad D(\mathbf{B})=C^1[0,l].$$

则(1.1)可写为:

$$\frac{d^2y}{dt^2}+\mathbf{B}\frac{dy}{dt}+\mathbf{A}y=0,\quad y(0)=y_0,\quad y'(0)=y_1.\tag{1.2}$$

考虑$C^1[0,l]\times C^1[0,l]$上的一阶抽象Cauchy问题:

$$\frac{d\varphi}{dt}=\mathcal{A}\varphi+\mathcal{B}\varphi,\quad \varphi(0)=\varphi_0=(y_0,y_1)^T,\tag{1.3}$$

其中

$$\mathcal{A}=\begin{pmatrix}0 & \mathbf{I}\\ -\mathbf{A} & 0\end{pmatrix},\quad \mathcal{B}=\begin{pmatrix}0 & 0\\ 0 & \mathbf{B}\end{pmatrix},$$

$$D(\mathcal{A})=D(\mathbf{A})\times C^1[0,l],\quad D(\mathcal{B})=C^1[0,l]\times C^1[0,l].$$

容易验证如果y是(1.2)的解,则$(y,y')^T$为(1.3)的解;反之,如果$(\varphi_1,\varphi_2)^T$是(1.3)的解,则φ_1为(1.2)的解且$\varphi_2=\frac{d\varphi_1}{dt}$. 因此得到

定理1 问题(1.2)与问题(1.3)等价.

对任意$\varphi=(\varphi_1,\varphi_2)^T\in C^1[0,l]\times C^1[0,l]$,定义范数:$\|\varphi\|_2=\max(\|\varphi_1\|_1,\|\varphi_2\|_1)$,则$C^1[0,l]\times C^1[0,l]$在$\|\cdot\|_2$下为一Banach空间.

定理2 算子$\mathcal{A}$在$C^1[0,l]\times C^1[0,l]$上生成三次积分半群$(S(t))_{t\geq 0}$且$\|S(t)\|_2\leq Me^{\omega t}$.

证明 由[5]知只需证明存在$\omega>0$,使得当$\mathrm{Re}\lambda>\omega>0$时,

$$\|(\lambda^2I+\mathbf{A})^{-1}\|_1\leq\frac{C}{|\lambda|},\quad C\text{ 为常数}$$

为了简便,仅就λ为大于零的实数进行讨论,对于λ为实部大于零的复数,除增加运算与估计的复杂性外没有本质出别.

对任意$f(x)\in C^1[0,l]$,考虑方程$(\lambda^2I+\mathbf{A})y(x)=f(x)$,运用常数变易法求得其解为:

$$y(x)=C_1e^{\mu_1x}+C_2e^{\mu_2x}+C_3e^{\mu_3x}+C_4e^{\mu_4x}-\int A_1e^{-\mu_1s}f(s)ds\cdot e^{\mu_1x}+$$
$$\int A_2e^{-\mu_2s}f(s)ds\cdot e^{\mu_2x}-\int A_3e^{-\mu_3s}f(s)ds\cdot e^{\mu_3x}+\int A_4e^{-\mu_4s}f(s)ds\cdot e^{\mu_4x},\tag{1.4}$$

其中

$$\mu_1=\lambda^{\frac12}e^{\frac{\pi}{4}i},\ \mu_2=\lambda^{\frac12}e^{\frac34\pi i},\ \mu_3=\lambda^{\frac12}e^{\frac54\pi i},\ \mu_4=\lambda^{\frac12}e^{\frac74\pi i},$$

$$A_1=\frac{1}{(\mu_4-\mu_1)(\mu_3-\mu_1)(\mu_2-\mu_1)},\quad A_2=\frac{1}{(\mu_4-\mu_2)(\mu_3-\mu_2)(\mu_2-\mu_1)},$$

$$A_3=\frac{1}{(\mu_4-\mu_3)(\mu_3-\mu_2)(\mu_3-\mu_1)},\quad A_4=\frac{1}{(\mu_4-\mu_3)(\mu_4-\mu_2)(\mu_4-\mu_1)},$$

$$C_1=\frac{D_1}{D},\quad C_2=\frac{D_2}{D},\quad C_3=\frac{D_3}{D},\quad C_4=\frac{D_4}{D},$$ 这里

$$D = [-8 + 2e^{\lambda^{\frac{1}{2}}2^{\frac{1}{2}}l} + 2e^{-\lambda^{\frac{1}{2}}2^{\frac{1}{2}}l} + 2e^{\lambda^{\frac{1}{2}}2^{\frac{1}{2}}li} + 2e^{-\lambda^{\frac{1}{2}}2^{\frac{1}{2}}li}]\lambda,$$

$$D_1 = G_1[(1+i)e^{\mu_4 l} - 2e^{\mu_3 l} + (1-i)e^{\mu_2 l}]\lambda - G_2[(\mu_4-\mu_2) + (\mu_2-\mu_3)e^{-\lambda^{\frac{1}{2}}2^{\frac{1}{2}}l} +$$
$$(\mu_3-\mu_4)e^{-\lambda^{\frac{1}{2}}2^{\frac{1}{2}}li}] + G_3[(\mu_4-\mu_3)e^{\mu_2 l} + (\mu_2-\mu_4)e^{\mu_3 l} + (\mu_3-\mu_2)e^{\mu_4 l}],$$

$$D_2 = G_1[(1+i)e^{\mu_1 l} - 2e^{\mu_4 l} + (1-i)e^{\mu_3 l}]\lambda + G_2[(\mu_1-\mu_3) + (\mu_4-\mu_1)e^{\lambda^{\frac{1}{2}}2^{\frac{1}{2}}l} +$$
$$(\mu_3-\mu_4)e^{-\lambda^{\frac{1}{2}}2^{\frac{1}{2}}li}] - G_3[(\mu_4-\mu_3)e^{\mu_1 l} + (\mu_3-\mu_1)e^{\mu_4 l} + (\mu_1-\mu_4)e^{\mu_3 l}],$$

$$D_3 = G_1[(1+i)e^{\mu_2 l} - 2e^{\mu_1 l} + (1-i)e^{\mu_4 l}]\lambda + G_2[(\mu_4-\mu_2) + (\mu_2-\mu_1)e^{\lambda^{\frac{1}{2}}2^{\frac{1}{2}}il} +$$
$$(\mu_1-\mu_4)e^{\lambda^{\frac{1}{2}}2^{\frac{1}{2}}l}] - G_3[(\mu_4-\mu_1)e^{\mu_2 l} + (\mu_2-\mu_4)e^{\mu_1 l} + (\mu_1-\mu_2)e^{\mu_4 l}],$$

$$D_4 = G_1[(1+i)e^{\mu_3 l} - 2e^{\mu_2 l} + (1-i)e^{\mu_1 l}]\lambda - G_2[(\mu_1-\mu_3) + (\mu_3-\mu_2)e^{-\lambda^{\frac{1}{2}}2^{\frac{1}{2}}l} +$$
$$(\mu_2-\mu_1)e^{\lambda^{\frac{1}{2}}2^{\frac{1}{2}}li}] + G_3[(\mu_3-\mu_1)e^{\mu_2 l} + (\mu_2-\mu_3)e^{\mu_1 l} + (\mu_1-\mu_2)e^{\mu_3 l}],$$

其中

$$G_1 = \int^l A_1 e^{-\mu_1 s} f(s)\,ds \cdot e^{\mu_1 l} - \int^l A_2 e^{-\mu_2 s} f(s)\,ds \cdot e^{\mu_2 l} + \int^l A_3 e^{-\mu_3 s} f(s)\,ds \cdot e^{\mu_3 l} -$$
$$\int^l A_4 e^{-\mu_4 s} f(s)\,ds \cdot e^{\mu_4 l},$$

$$G_2 = (A_1 - A_2 + A_3 - A_4) f(0),$$

$$G_3 = (A_1 - A_2 + A_3 - A_4) f(l) + \int^l A_1 \mu_1 e^{-\mu_1 s} f(s)\,ds \cdot e^{\mu_1 l} - \int^l A_2 \mu_2 e^{-\mu_2 s} f(s)\,ds \cdot e^{\mu_2 l} +$$
$$\int^l A_3 \mu_3 e^{-\mu_3 s} f(s)\,ds \cdot e^{\mu_3 l} - \int^l A_4 \mu_4 e^{-\mu_4 s} f(s)\,ds \cdot e^{\mu_4 l}.$$

如果能证明存在 $\omega > 0$, **使得当** $\lambda > \omega$ **时** $\|y\|_1 = \|y\| + \|y''\| \le \frac{C}{\lambda}\|f\|_1$ (C **为常数**), **这就是** $\|(\lambda^2 I + \mathbf{A})^{-1}\|_1 \le \frac{C}{\lambda}$, **从而就证明了此定理　为此, 先看** $\|y\|$.

注意到 $|e^{\mu_1 s}| = e^{2^{-\frac{1}{2}}\lambda^{\frac{1}{2}}s} \ge 1$, $|e^{\mu_2 s}| = e^{2^{-\frac{1}{2}}\lambda^{\frac{1}{2}}s} \le 1$, $|e^{\mu_3 s}| = e^{2^{-\frac{1}{2}}\lambda^{\frac{1}{2}}s} \le 1$, $|e^{\mu_4 s}| = e^{2^{-\frac{1}{2}}\lambda^{\frac{1}{2}}s} \ge 1$. **考虑**(1. 4)**中的第六项与第七项, 即**

$$\int A_2 e^{-\mu_2 s} f(s)\,ds \cdot e^{\mu_2 x}, \quad - \int A_3 e^{-\mu_3 s} f(s)\,ds \cdot e^{\mu_3 l}.$$

$$\left|A_2 \int e^{-\mu_2 s} f(s)\,ds \cdot e^{\mu_2 x}\right| \le |A_2| \left|\int |e^{-\mu_2 s}|\,ds \cdot \|f\| \cdot |e^{\mu_2 x}|\right|$$
$$\le |A_2| \left|\int e^{\lambda^{\frac{1}{2}}2^{-\frac{1}{2}}s}\,ds \cdot e^{-\lambda^{\frac{1}{2}}2^{-\frac{1}{2}}x} \cdot \|f\| \le |A_2| \frac{1}{\lambda^{\frac{1}{2}}2^{-\frac{1}{2}}}\|f\| \le \frac{C}{\lambda^2}\|f\|,\right.$$

这里 C **为常数　在以后的估计中为了方便, 凡是出现常数的地方, 均用** C **表示, 尽管它们彼此不同**

$$\left|-A_3\int e^{-\mu_3 s}f(s)\,\mathrm{d}s\cdot e^{\mu_3 x}\right|\leq |A_3|\cdot\|f\|\int e^{\lambda^{\frac{1}{2}}2^{-\frac{1}{2}}s}\mathrm{d}s\cdot e^{-\lambda^{\frac{1}{2}}2^{-\frac{1}{2}}s}\leq\frac{C}{\lambda^2}\|f\|$$

现在考虑(1.4)**中的**$C_2e^{\mu_2 x}$**和**$C_3e^{\mu_3 x}$. **首先考虑**$C_2e^{\mu x}$, $|C_2e^{\mu_2 x}|\leq|C_2|\cdot|e^{\mu_2 x}|\leq|C_2|$. **注意到**

$$\left|\int A_1e^{-\mu_1 s}f(s)\,\mathrm{d}s\cdot e^{\mu_1 l}\right|\leq\frac{C}{\lambda^2}e^{2^{-\frac{1}{2}}\lambda^{\frac{1}{2}}l}\|f\|,$$

$$\left|\int A_4e^{-\mu_4 s}f(s)\,\mathrm{d}s\cdot e^{\mu_4 l}\right|\leq\frac{C}{\lambda^2}e^{2^{-\frac{1}{2}}\lambda^{\frac{1}{2}}l}\|f\|,$$

所以

$$|G_1|\leq\frac{C'}{\lambda^2}e^{2^{-\frac{1}{2}}\lambda^{\frac{1}{2}}l}\|f\|+\frac{C''}{\lambda^2}\|f\|\leq\frac{C}{\lambda^2}e^{2^{-\frac{1}{2}}\lambda^{\frac{1}{2}}l}\|f\|,$$

$$\left|[(1+i)e^{\mu_2 l}-2e^{\mu_1 l}+(1-i)e^{\mu_4 l}]\lambda\right|\leq C\lambda e^{2^{-\frac{1}{2}}\lambda^{\frac{1}{2}}l},$$

$$|G_2|\leq\frac{C}{\lambda^{\frac{3}{2}}}\|f\|,\qquad|G_3|\leq\frac{C}{\lambda^{\frac{3}{2}}}e^{\lambda^{\frac{1}{2}}2^{\frac{1}{2}}l}\|f\|,$$

$$\left|(\mu_4-\mu_2)+(\mu_2-\mu_1)e^{\lambda^{\frac{1}{2}}2^{\frac{1}{2}}il}+(\mu_1-\mu_4)e^{\lambda^{\frac{1}{2}}2^{\frac{1}{2}}l}\right|\leq C\lambda^{\frac{1}{2}}e^{\lambda^{\frac{1}{2}}2^{\frac{1}{2}}l}$$

$$\left|(\mu_4-\mu_3)e^{\mu_1 l}+(\mu_3-\mu_1)e^{\mu_4 l}+(\mu_1-\mu_4)e^{\mu_3 l}\right|\leq C\lambda^{\frac{1}{2}}e^{\frac{2^{\frac{1}{2}}}{2}\lambda^{\frac{1}{2}}l}.$$

从而

$$|C_2e^{\mu_2 l}|\leq|C_2|\leq\left|\frac{D_2}{D}\right|\leq\frac{1}{|D|}[C\frac{1}{\lambda}e^{2^{\frac{1}{2}}\lambda^{\frac{1}{2}}l}\|f\|+\frac{C}{\lambda}e^{\sqrt{2}\lambda^{\frac{1}{2}}l}\|f\|+\frac{C}{\lambda}e^{2^{\frac{1}{2}}\lambda^{\frac{1}{2}}}l\|f\|]$$

$$\leq\frac{C}{\lambda}e^{2^{\frac{1}{2}}\lambda^{\frac{1}{2}}l}\|f\|\cdot\frac{1}{|D|}.$$

又因为

$$e^{-2^{\frac{1}{2}}\lambda^{\frac{1}{2}}l}\cdot|D|=\left|-8e^{-2^{\frac{1}{2}}\lambda^{\frac{1}{2}}l}+2+2e^{-2^{\frac{1}{2}}\lambda^{\frac{1}{2}}l}\right.$$

$$\left.+2e^{-2^{\frac{1}{2}}\lambda^{\frac{1}{2}}l+2^{\frac{1}{2}}\lambda^{\frac{1}{2}}li}+2e^{-2^{\frac{1}{2}}\lambda^{\frac{1}{2}}l-2^{\frac{1}{2}}\lambda^{\frac{1}{2}}li}\right|>1,$$

所以 $|C_2e^{\mu_2 x}|\leq\frac{C}{\lambda^2}\|f\|$. **同理可证** $|C_3e^{\mu_3 x}|\leq\frac{C}{\lambda^2}\|f\|$.

下面估计(1.4)**中的**$C_1{}^{\mu_1 x}-\int A_1e^{-\mu_1 s}f(s)\,\mathrm{d}s\cdot e^{\mu_1 x}$. **由于**

$$C_1e^{\mu_1 x}-\int A_1e^{-\mu_1 s}f(s)\,\mathrm{d}s\cdot e^{\mu_1 x}=\frac{D_1}{D}e^{\mu_1 x}-\int A_1e^{-\mu_1 s}f(s)\,\mathrm{d}s\cdot e^{\mu_1 x}.$$

注意到

$$\left|\frac{G_2[(\mu_4-\mu_2)+(\mu_2-\mu_3)e^{-2^{\frac{1}{2}}\lambda^{\frac{1}{2}}l}+(\mu_3-\mu_4)e^{-2^{\frac{1}{2}}\lambda^{\frac{1}{2}}li}]}{D}\cdot e^{\mu_1 x}\right|\leq\frac{C}{\lambda^2}\|f\|,$$

$$\left|\frac{[-\int A_2e^{-\mu_2 s}f(s)\,ds\cdot e^{\mu_2 l}+\int A_3e^{-\mu_3 s}f(s)\,\mathrm{d}s\cdot e^{\mu_3 l}]\cdot[(1+i)e^{\mu_4 l}-2e^{\mu_3 l}+(1-i)e^{\mu_2 l}]\lambda}{D}\right|$$

$$\cdot \left| e^{\mu_1 x} \right| \le \frac{C}{\lambda^2} \| f \|,$$

$$\left| \frac{(A_1 - A_2 + A_3 - A_4) f(l) [(\mu_4 - \mu_3) e^{\mu_2 l} + (\mu_2 - \mu_4) e^{\mu_3 l} + (\mu_3 - \mu_2) e^{\mu_4 l}]}{D} \right|$$

$$\cdot \left| e^{\mu_1 x} \right| \le \frac{C}{\lambda^2} \| f \|,$$

$$\left| \frac{[- \int^l A_2 \mu_2 e^{-\mu_2 s} f(s) \mathrm{d}s \cdot e^{\mu_2 l} + \int^l A_3 \mu_3 e^{-\mu_3 s} f(s) \mathrm{d}s \cdot e^{\mu_3 l}]}{D} \right|$$

$$\cdot \left| [(\mu_4 - \mu_3) e^{\mu_2 l} + (\mu_2 - \mu_4) e^{\mu_3} + (\mu_3 - \mu_2) e^{\mu_4 l}] \cdot e^{\mu_1 x} \right| \le \frac{C}{\lambda^2} \| f \|$$

由于

$$[\int^l A_1 e^{-\mu_1 s} f(s) \mathrm{d}s \cdot e^{\mu_1 l} - \int^l A_4 e^{-\mu_4 s} f(s) \mathrm{d}s \cdot e^{\mu_4 l}][(1+i) e^{\mu_4 l} - 2 e^{\mu_3 l} + (1-i) e^{\mu_2 l}] \lambda \cdot e^{\mu_1 x} +$$

$$[\int^l A_1 \mu_1 e^{-\mu_1 s} f(s) \mathrm{d}s \cdot e^{\mu_1 l} - \int^l A_4 \mu_4 e^{-\mu_4 s} f(s) \mathrm{d}s \cdot e^{\mu_4 l}] \cdot [(\mu_4 - \mu_3) e^{\mu_2 l} + (\mu_2 - \mu_4) e^{\mu_3 l} +$$

$$(\mu_3 - \mu_2) e^{\mu_4 l}] e^{\mu_1 x} - D \int^x A_1 e^{-\mu_1 s} f(s) \mathrm{d}s \cdot e^{\mu_1 x}$$

$$= 2\lambda \int^l A_1 e^{-\mu_1 s} f(s) \mathrm{d}s \cdot e^{\mu_1 l} \cdot e^{\mu_4 l} \cdot e^{\mu_1 x} - 4\lambda \int^l A_1 e^{-\mu_1 s} f(s) \mathrm{d}s \cdot e^{\mu_1 l} \cdot e^{\mu_3 l} \cdot e^{\mu_1 x} +$$

$$2\lambda \int^l A_1 e^{-\mu_1 s} f(s) \mathrm{d}s \cdot e^{\mu_1 l} \cdot e^{\mu_2 l} \cdot e^{\mu_1 x} + 2\lambda(1-i) \int^l A_4 e^{-\mu_4 s} f(s) \mathrm{d}s \cdot e^{\mu_4 l} \cdot e^{\mu_3 l} \cdot e^{\mu_1 x} -$$

$$2\lambda(1-i) \int^l A_4 e^{-\mu_4 s} f(s) \mathrm{d}s \cdot e^{\mu_4 l} \cdot e^{\mu_2 l} \cdot e^{\mu_1 x} - D \int^x A_1 e^{-\mu_1 (s-x)} f(s) \mathrm{d}s \qquad (1.5)$$

因为 $\left| \frac{1}{D} [(1.5)$ **中的第2项+ 第3项+ 第4项+ 第5项**$] \right| \le \frac{C}{\lambda^2} \| f \|$.

下面来看(1.5)**中的第1项与第6项**:

$$2\lambda \int^l A_1 e^{-\mu_1 s} f(s) \mathrm{d}s \cdot e^{\mu_1 l} \cdot e^{\mu_4 l} \cdot e^{\mu_1 x} - D \int^x A_1 e^{-\mu_1 (s-x)} f(s) \mathrm{d}s$$

$$= 2\lambda \int^l A_1 e^{-\mu_1 s} f(s) \mathrm{d}s \cdot e^{\mu_1 l} \cdot e^{\mu_4 l} \cdot e^{\mu_1 x} - [D - 2\lambda e^{\lambda^{\frac{1}{2}} 2^{\frac{1}{2}} l}] \cdot \int^x A_1 e^{-\mu_1 s} f(s) \mathrm{d}s \cdot e^{\mu_1 x} -$$

$$2\lambda e^{-\lambda^{\frac{1}{2}} 2^{\frac{1}{2}} l} \int^l A_1 e^{-\mu_1 s} f(s) \mathrm{d}s \cdot e^{\mu_1 x},$$

注意到

$$\left| \frac{D - 2\lambda e^{\lambda^{\frac{1}{2}} 2^{\frac{1}{2}} l}}{D} \int^x A_1 e^{-\mu_1 s} f(s) \mathrm{d}s \cdot e^{\mu_1 x} \right| \le \frac{C}{\lambda^2} \| f \|,$$

$$2\lambda \int^l A_1 e^{-\mu_1 s} f(s) \mathrm{d}s \cdot e^{\mu_1 l} \cdot e^{\mu_4 l} \cdot e^{\mu_1 x} - 2\lambda e^{\lambda^{\frac{1}{2}} 2^{\frac{1}{2}} l} \int^x A_1 e^{-\mu_1 s} f(s) \mathrm{d}s \cdot e^{\mu_1 x}$$

$$= 2\lambda e^{\lambda^{\frac{1}{2}} 2^{\frac{1}{2}} l} \int^l A_1 e^{-\mu_1 (s-x)} f(s) \mathrm{d}s - 2\lambda e^{\lambda^{\frac{1}{2}} 2^{\frac{1}{2}} l} \int^x A_1 e^{-\mu_1 (s-x)} f(s) \mathrm{d}s$$

$$= 2\lambda e^{\lambda^{\frac{1}{2}} 2^{\frac{1}{2}} l} \int_x^l A_1 e^{-\mu_1 (s-x)} f(s) \mathrm{d}s$$

$$= 2\lambda e^{\lambda^{\frac{1}{2}} 2^{\frac{1}{2}} (l - x)} \int A_1 e^{-\mu_1 s} f(s + x)\,ds,$$

$$\left| \frac{2\lambda e^{\lambda^{\frac{1}{2}} 2^{\frac{1}{2}} (l - x)} \int A_1 e^{-\mu_1 s} f(s + x)\,ds}{D} \right| \le \frac{C}{\lambda^2} \|f\|$$

至此证明了(1.4)**中的** $\left| C_1 e^{\mu_1 x} - \int A_1 e^{-\mu_1 s} f(s)\,ds \cdot e^{\mu_1 x} \right| \le \frac{C}{\lambda^2} \|f\|$ **类似的方法可以证明**(1.4)**中的** $\left| C_4 e^{\mu_4 x} + \int A_4 e^{-\mu_4 s} f(s)\,ds \cdot e^{\mu_4 x} \right| \le \frac{C}{\lambda^2} \|f\|$ **从而证明了**: $\|y(x)\| \le \frac{C}{\lambda^2} \|f\|$. **下面来看** $|y'(x)|$. **由** $y'(x)$ **的表达式及对** $\|y\|$ **的估计得到** $\|y'(x)\| \le \frac{C}{\lambda^{\frac{3}{2}}} \|f\|$ **设** $\lambda > 1$, **则**

$$\|y(x)\| \le \frac{C}{\lambda^2} \|f\| \le \frac{C}{\lambda} \|f\|, \qquad \|y'(x)\| \le \frac{C}{\lambda^{\frac{3}{2}}} \|f\| \le \frac{C}{\lambda} \|f\|$$

从而 $\|y\| + \|y'\| = \|y\|_1 \le \frac{C}{\lambda} \|f\| \le \frac{C}{\lambda} (\|f\| + \|f'\|)$. **因此** $\|(\lambda^2 I + \mathbf{A})^{-1}\|_1 \le \frac{C}{\lambda}$. **于是证明了对任意** $\mathrm{Re}\lambda > 0$, **存在** $\omega > 0$, **使得当** $\mathrm{Re}\lambda > \omega > 0$ **时**, $\|(\lambda^2 I + \mathbf{A})^{-1}\|_1 \le \frac{C}{|\lambda|}$. □

由定理2**及**[5]**中的定理**5.2**得**:

(1) **设** $\|\varphi\|_F = \sup_{t \ge 0} \|e^{-\omega t} S^{(3)}(t) \varphi\|_2$, **则** $\|\cdot\|_F$ **在** $D(\mathbf{A}^3)$ **上定义了一个范数且满足** $\|\varphi\|_2 \le \|\varphi\|_F$.

(2) **设** $F = \overline{D(\mathbf{A}^6)}^{\|\cdot\|_F} = \overline{D(\mathbf{A}^3) \times D(\mathbf{A}^3)}^{\|\cdot\|_F}$. **定义**: $\mathbf{A}_F \varphi(x) = \mathbf{A}\varphi(x)$, **对任意** $\varphi(x) \in D(\mathbf{A}_F)$, **其中**

$$D(\mathbf{A}_F) = \{\varphi(x) \in D(\mathbf{A}) \cap F : \mathbf{A}\varphi(x) \in F\}$$
$$= \{\varphi(x) \in D(\mathbf{A}) \times C^1[0, l] \cap \overline{D(\mathbf{A}^3) \times D(\mathbf{A}^3)}^{\|\cdot\|_F} : \mathbf{A}\varphi(x) \in \overline{D(\mathbf{A}^3) \times D(\mathbf{A}^3)}^{\|\cdot\|_F}\}.$$

则 $\mathbf{A}_F$ **在** F **上生成强连续半群** $T(t) = S^{(3)}(t)|_F$. **如果** $\mathbf{B}_F$ (**即** $\mathbf{B}_F$ **在** F **上的限制**) **为** $F \to F$ **上的有界线性算子**, **则根据强连续半群的扰动定理知**: $\mathbf{A}_F + \mathbf{B}_F$ **在** F **上生成强连续半群**. **从而对任意** $\varphi_0 \in D(\mathbf{A}_F)$, **问题**(1.3) **在** $D(\mathbf{A}_F)$ **上存在唯一解**. **为此**, **设** $b(x) \in D(\mathbf{A}^3)$, **则** $\mathbf{B}$ **将** F **映到** F, **只需证明** $\mathbf{B}_F$ **为** F **上的有界线性算子**. **事实上**, **对任意** $\varphi \in F$, $\varphi = (\varphi_1, \varphi_2)^T$,

$$\|\mathbf{B}\varphi\|_F = \sup_{t \ge 0} \|e^{-\omega t} S^{(3)}(t) \mathbf{B}\varphi\|_2 = \sup_{t \ge 0} \left\| e^{-\omega t} S^{(3)}(t) \begin{pmatrix} 0 & 0 \\ 0 & -\mathbf{B} \end{pmatrix} \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} \right\|_2$$
$$= \sup_{t \ge 0} \left\| e^{-\omega t} S^{(3)}(t) \begin{pmatrix} 0 \\ -\mathbf{B}\varphi_2 \end{pmatrix} \right\|_2 = \sup_{t \ge 0} \left\| e^{-\omega t} S^{(3)}(t) \begin{pmatrix} 0 \\ b(x) \sum_{i=1}^{n} \varphi_2'(x_i) \end{pmatrix} \right\|_2$$
$$= \left| \sum_{i=1}^{n} \varphi_2'(x_i) \right| \sup_{t \ge 0} \left\| e^{-\omega t} S^{(3)}(t) \begin{pmatrix} 0 \\ b(x) \end{pmatrix} \right\|_2 \le \sum_{i=1}^{n} |\varphi_2'(x_i)| \cdot M \le \sum_{i=1}^{n} \|\varphi_2\|_1 \cdot M$$
$$= nM \|\varphi_2\|_1 \le nM \|\varphi\|_2 \le nM \|\varphi\|_F,$$

这里

$$M = \sup_{t \ge 0} \left\| e^{-\omega t} S^{(3)}(t) \begin{pmatrix} 0 \\ b(x) \end{pmatrix} \right\|_2$$

为常数. **从而**, $\|\mathbf{B}\|_F \le nM$. **由以上分析及定理**1**得到**:

定理3 设 $b(x) \in D(\mathbf{A}^3)$，则对任意 $(y_0(x), y_1(x))^T \in D(\mathbf{A}_F)$，问题(1.1)存在唯一解 $y(x,t) \in \overline{D(\mathbf{A}^3)}^{\|\cdot\|_F} \subset C^1[0,l]$.

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Existence of Classical Solution for Feedback Control System of Elastic Beam Vibration with Point Observation

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Abstract

This paper discusses the elastic beam vibration system with two sides fixed. By taking the angular velocity measured in some points on beam as the feedback control rule, the existence and uniqueness of classical solution is proved by means of the integrated semigroups theory of bounded linear operators.

Keywords elastic beam vibration system, integrated semigroup, classical solution.