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Approximation of Fixed Point and Solution for φ-Hemicontraction and φ-Strongly Quasi-Accretive Operator without Lipschitz Assumption *

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Abstract: Let X be a real uniformly smooth Banach space and let $T : D(T) \subset X \to X$ be φ–hemicontractive and locally bounded at its fixed point $q \in F(T)$. Under some suitable assumptions on the iteration parameters $\{\alpha_n\}$ and $\{\beta_n\}$, we have proved that the Mann and Ishikawa iteration processes for T converge strongly to the unique fixed point q of T. Several related results deal with iterative solutions of nonlinear equations involving φ–strongly quasi-accretive operators. Our results extend and generalize those corresponding ones by Xu and Roach, Zhou and Jia and others.

Key words: φ–strong pseudocontraction; φ–strongly accretive operator; Ishikawa iteration process; Reich's inequality.

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1. Introduction

Let X be a real Banach space with dual X^*. The normalized duality mapping from X to the family of subsets of X^* is defined by

$$Jx = \{x^* \in X^* : \langle x, x^* \rangle = ||x||.||x^*||, ||x^*|| = ||x||\},$$

where $\langle \cdot, \cdot \rangle$ denotes the generalized duality pairing. It is well known that if X^* is strictly convex, then J is single-valued and $J(tx) = tJx$, for all $t \ge 0$; if X is uniformly smooth, then J is uniformly continuous on bounded subsets of X. We denote the single-valued normalized duality mapping by j.

A mapping T with domain $D(T)$ in X is said to be accretive if for each $x, y \in D(T)$ there exists $j(x-y) \in J(x-y)$ such that

$$\langle Tx - Ty, j(x-y) \rangle \ge 0. \tag{1}$$

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Furthermore, T is called strongly accretive if there exists a constant $k > 0$ such that

$$\langle Tx - Ty, j(x-y)\rangle \geq k||x-y||^2. \tag{2}$$

T is said to be φ-strongly accretive if there exists a strictly increasing function $\varphi : [0,\infty) \to [0,\infty)$ with $\varphi(0) = 0$ such that the inequality

$$\langle Tx - Ty, j(x-y)\rangle \geq \varphi(||x-y||)||x-y|| \tag{3}$$

holds for every $x, y \in D(T)$ and some $j(x-y) \in J(x-y)$.

Let $N(T) = \{x \in E : Tx = 0\}$. If $N(T) \neq \emptyset$, and the inequalities (1),(2) and (3) hold for any $x \in D(T)$ but $y \in N(T)$, then the corresponding operator T is called quasi-accretive, strongly quasi-accretive and $\varphi-$strongly quasi-accretive, respectively. T is said to be a quasi-pseudocontraction, strongly quasi- pseudocontraction, and φ-hemicontraction if $(I-T)$ is quasi-accretive, strongly quasi-accretive,and $\varphi-$strongly quasi-accretive, respectively. T is said to be a pseudocontraction, strong pseudocontraction, and $\varphi-$strong pseudocontraction if and only if $(I-T)$ is accretive, strongly accretive and $\varphi-$strongly accretive, respectively, where I denotes the identity operator on X. Let $F(T) = \{x \in D(T) : Tx = x\}$. T is said to be a quasi-pseudocontraction, strong quasi-pseudocontraction and $\varphi-$hemicontraction if $(I-T)$ is quasi-accretive, strongly quasi-accretive and $\varphi-$strongly quasi-accretive, respectively. Such operators have been studied and used extensively by several researchers (see, e.g.,[12,14, 16, 18]).

Recently, Zhou and Jia[16], and Zhou[17] established several convergence results concerning iterative solution involving nonlinear accretive-type operator equations in uniformly smooth Banach spaces without Lipschitz assumption. These results extended and generalized many known ones. Actually, the main ideas and methods used in the proof of Zhou and Jia[16] are still effective for the case where the accretive operator A is only defined on a closed ball in X.

An operator $T : D(T) \subset X \to X$ is called locally bounded at some $x_0 \in D(T)$ if there exists a closed ball $B_r(x_0) \subset D(T)$ such that $T(B_r(x_0))$ is bounded. Accordingly, T is said to be locally bounded if T is locally bounded at every $x \in D(T)$. It is well known that if X is a uniformly smooth Banach space and $T : D(T) \subset X \to X$ is an accretive operator with an open domain $D(T)$. Then T is locally bounded.

It is our purpose in this paper to localize the main results of Zhou and Jia[16] by using Reich's inequality instead of Xu and Roach's inequality. For this purpose, we need the following known result due to Reich[13].

Lemma 1.1 *Let X be a real uniformly smooth Banach space. Then there exists a nondecreasing continuous function $b : [0,\infty) \to [0,\infty)$ satisfying*

(i) $b(ct) \leq cb(t)$ *for all* $c \geq 1$;

(ii) $\lim_{t\to 0^+} b(t) = 0$;

(iii) $||x+y||^2 \leq ||x||^2 + 2\langle y, j(x)\rangle + \max\{||x||, 1\}||y||b(||y||)$ *for all* $x, y \in X$.

The inequality (iii) is called Reich's inequality.

In the sequel, we always assume that X is a real uniformly smooth Banach space.

2. Main Results

Theorem 2.1 *Let $T : D(T) \subset X \to X$ be a $\varphi-$hemicontraction. Suppose T is locally bounded at its fixed point $q \in F(T)$. Then there exists a closed ball B_1 such that the Ishikawa iterative sequence $\{x_n\}_{n\geq 0}$ defined by*

$$(\mathrm{IS})_1 \begin{cases} x_0 \in B_1 \\ x_{n+1} = (1-\alpha_n)x_n + \alpha_n T y_n, n \geq 0, \\ y_n = (1-\beta_n)x_n + \beta_n T x_n, n \geq 0, \end{cases}$$

remains in B_1 and converges strongly to q provided that $\{\alpha_n\}$ and $\{\beta_n\}$ are two real sequences in $[0,1)$ satisfying the following conditions:

(i) $\alpha_n \leq \min\{\tau, \frac{r}{4M}\}, n \geq 0$;

(ii) $\beta_n \leq \min\{\tau, \frac{\delta}{M}, \frac{r}{4M}\}, n \geq 0$;

(iii) $\sum_{n=0}^{\infty} \alpha_n = \infty$;

(iv) $\alpha_n \to 0$, $\beta_n \to 0$ *as* $n \to \infty$, *where* r, τ, M *and* δ *are some fixed constants.*

Proof Since $T : D(T) \subset X \to X$ is locally bounded at $q \in F(T)$, we can choose $r > 0$ such that $B_1 = \{x \in X : \|x - q\| \leq r\}$ is contained in $D(T)$ and $T(B_1)$ is bounded. Let $M = \sup\{\|Ty - y\| : y \in B_1\} + 1$ and $M_1 = \max\{r, 1\}(M + r)^2$. Since $b : [0,\infty) \to [0,\infty)$ is continuous and nondecreasing, we can choose the largest positive number τ such that $\frac{b(\tau)}{1-\tau} \leq \frac{r\varphi(\frac{r}{4})}{4M_1}$. This is possible, because $\frac{b(t)}{1-t} \to 0$ as $t \to 0$. Since X is uniformly smooth, j is uniformly continuous on bounded subsets of X. Consequently, for given $\varepsilon = \frac{r\varphi(\frac{r}{4})}{8(M+r)}$, we can also choose $\delta > 0$ such that $\|jx - jy\| \leq \varepsilon$ whenever $x, y \in B_r(0)$ and $\|x - y\| \leq \delta$. Now we are in a good position to choose $\{\alpha_n\}$ and $\{\beta_n\}$ satisfying conditions (i) and (ii). We finish the proof of Theorem 2.i by the following two steps.

Step I. We shall prove that the sequence $\{x_n\}$ defined by $(\mathrm{IS})_1$ is well-defined and remains in B_1. To this end, we show first that $y_n \in B_1$ whenever $x_n \in B_1$. Let $x_n \in B_1$, then $\|x_n - q\| \leq r$, and this implies that $\|Tx_n - x_n\| \leq M$.

By using Reich's inequality and $(\mathrm{IS})_1$, we have

$$\begin{aligned} \|y_n - q\|^2 =& \|(1-\beta_n)(x_n - q) + \beta_n(Tx_n - q)\|^2 \\ \leq& (1-\beta_n)^2\|x_n - q\|^2 + 2\beta_n(1-\beta_n)\langle Tx_n - q, j(x_n - q)\rangle + \\ & \max\{\|x_n - q\|, 1\}\beta_n\|Tx_n - q\| b(\beta_n\|Tx_n - q\|) \\ \leq& [(1-\beta_n)^2 + 2\beta_n(1-\beta_n)]r^2 - \\ & 2\beta_n(1-\beta_n)\varphi(\|x_n - q\|)\|x_n - q\| + M_1\beta_n b(\beta_n) \\ \leq& r^2 - 2\beta_n(1-\beta_n)\varphi(\|x_n - q\|)\|x_n - q\| + M_1\beta_n b(\beta_n). \end{aligned} \tag{4}$$

Assume that $\|y_n - q\| > r$, then

$$\|x_n - q\| \geq r - \beta_n\|Tx_n - x_n\| \geq r - \beta_n M \geq \frac{r}{2}, \tag{5}$$

and hence

$$\varphi(\|x_n - q\|) \geq \varphi(\frac{r}{2}). \tag{6}$$

Combining (5)–(6) with (4), we have $\|y_n - q\| \leq r$, which is in contradiction with the assumption that $\|y_n - q\| > r$. This contradiction shows $y_n \in B_1$ whenever $x_n \in B_1$.

Let $x_n \in B_1$, then $y_n \in B_1$ by the above argument. It follows that $\|x_n - Ty_n\| \leq \|x_n - y_n\| + \|y_n - Ty_n\| \leq 2M$.

Set $a_n = \|j(x_n - q) - j(y_n - q)\|$. Then

$$a_n \leq \frac{r\varphi(\frac{r}{4})}{8(M+r)}, \tag{7}$$

since $\|x_n - y_n\| \leq \beta_n M \leq \delta$.

Now we want to prove that $x_{n+1} \in B_1$. If it is not the case, assume that $\|x_{n+1} - q\| > r$. Then

$$\|x_n - q\| \geq r - 2\alpha_n M \geq \frac{r}{2}, \tag{8}$$

and so

$$\|y_n - q\| \geq \frac{r}{2} - \beta_n M \geq \frac{r}{4}, \tag{9}$$

which implies that $\varphi(\|y_n - q\|) \geq \varphi(\frac{r}{4})$.

Again using Reich's inequality, $(\mathrm{IS})_1$, and (7)-(9), we have

$$\begin{aligned}\|x_{n+1} - q\|^2 =& \|(1-\alpha_n)(x_n - q) + \alpha_n(Ty_n - q)\|^2 \\ \leq& (1-\alpha_n)^2\|x_n - q\|^2 + 2\alpha_n(1-\alpha_n)a_n(M+r) + \\ & 2\alpha_n(1-\alpha_n)\|y_n - q\|^2 - \\ & 2\alpha_n(1-\alpha_n)\varphi(\|y_n - q\|)\|y_n - q\| + M_1\alpha_n b(\alpha_n) \\ \leq& r^2,\end{aligned} \tag{10}$$

which implies that $\|x_{n+1} - q\| \leq r$, a contradiction. This contradiction shows that $x_{n+1} \in B_1$ whenever $x_n \in B_1$. By induction, we see that $x_n \in B_1$ for all $n \geq 0$. Consequently, we have $\|Tx_n - x_n\| \leq M$ and $\|Ty_n - q\| \leq (M+r)$ for all $n \geq 0$.

Observe that

$$\|y_n - x_n\| \leq \beta_n M \to 0, \tag{11}$$

as $n \to \infty$, we assert that $a_n \to 0$ as $n \to \infty$. by the uniform continuity of j.

Step II. We shall prove that $x_n \to q$ as $n \to \infty$. By the argument above, we have

$$\|x_{n+1} - q\|^2 \leq \|x_n - q\|^2 - 2\alpha_n(1-\alpha_n)\varphi(\|y_n - q\|)\|y_n - q\| + o(\alpha_n). \tag{12}$$

Set $a = \liminf\{\|y_n - q\| : n \geq 0\}$. Then $a = 0$. If not, assume that $a > 0$. Then there exists some N such that $\|y_n - q\| \geq \frac{a}{2}$ for all $n \geq N$, and hence $\varphi(\|y_n - q\|) \geq \varphi(\frac{a}{2}) > 0$ for all $n \geq N$. At this point, we can choose $n_0 \geq N$ so large that

$$o(\alpha_n) < \alpha_n(1-\alpha_n)\varphi(\frac{a}{2})\frac{a}{2}, \tag{13}$$

for all $n \geq n_0$. It follows from (12) and (13) that

$$\frac{a}{2}\varphi(\frac{a}{2})\sum_{n\geq n_0}^{\infty}\alpha_n(1-\alpha_n) \leq \|x_{n_0} - q\|^2, \tag{14}$$

i.e., a contradiction. Consequently, $\liminf_{n\to\infty}\|y_n - q\| = 0$. Hence there exists a subsequence $\{y_{n_j}\}$ of $\{y_n\}$ such that $y_{n_j} \to q$ as $j \to \infty$. It follows from $(\mathrm{IS})_1$ that $x_{n_j} \to q$ as $j \to \infty$. Again using $(\mathrm{IS})_1$, we can prove that for arbitrary $\varepsilon > 0$, there exists some fixed $n_j \geq 0$ such that $\|x_{n_j+m} - q\| \leq \varepsilon$, $\forall m \geq 0$. This implies that $x_n \to q$ as $n \to \infty$. We complete the proof of Theorem 2.1. □

Corollary 2.1 *Let $T : D(T) \subset X \to X$ be a $\varphi-$strong pseudocontraction with an open domain $D(T)$. Suppose T has a fixed point $q \in F(T)$. Let $\{\alpha_n\}$ and $\{\beta_n\}$ be as in Theorem 2.1. Then the conclusion of Theorem 2.1 holds true.*

Proof We observe that $(I-T)$ is φ-strongly accretive and hence is accretive. Since $D(T)$ is an open subset of X, we see that $(I-T)$ is locally bounded at $q \in D(T)$. Consequently, T is locally bounded at $q \in D(T)$ and $\varphi-$hemicontractive. Now the conclusion follows from Theorem 2.1. □

Corollary 2.2 *Let $T : D(T) \subset X \to X$ be a $\varphi-$hemicontraction. Suppose φ is surjective and $D(T)$ contains a closed ball $B_2 = \{x \in D(T) : \|x - x_0\| \leq 3\varphi^{-1}(\|x_0 - Tx_0\|)\}$ such that $T(B_2)$ is bounded for some initial value $x_0 \in D(T)$. Then the Ishikawa iterative sequence $\{x_n\}_{n\geq 0}$ generated by*

$$(\mathrm{IS})_2 \begin{cases} x_0 \in B_1 \\ x_{n+1} = (1-\alpha_n)x_n + \alpha_n T y_n, n \geq 0, \\ y_n = (1-\beta_n)x_n + \beta_n T x_n, n \geq 0, \end{cases}$$

remains in B_2 and converges strongly to the unique fixed point of T provided that $\{\alpha_n\}$ and $\{\beta_n\}$ are two real sequences in $[0,1)$ satisfying the following conditions:

(i) $\alpha_n \leq \min\{\tau, \frac{\varphi^{-1}(\|x_0 - Tx_0\|)}{4M}\}, n \geq 0;$

(ii) $\beta_n \leq \min\{\tau, \frac{\delta}{2M}, \frac{\varphi^{-1}(\|x_0 - Tx_0\|)}{4M}\}, n \geq 0;$

(iii) $\sum_{n=0}^{\infty} \alpha_n = \infty;$

(iv) $\alpha_n \to 0$, $\beta_n \to 0$ *as* $n \to \infty$, *where* τ, M *and* δ *are some fixed constants.*

Proof Since T is $\varphi-$hemicontractive, we know that T has a unique fixed point in $D(T)$, say, q. Set $r = 2\varphi^{-1}(\|x_0 - Tx_0\|)$. Then $B_r = \{x \in D(T) : \|x - q\| \leq r\}$ is contained in B_2. Now the conclusion of Corollary 2.2 follows from Theorem 2.1. □

Theorem 2.2 *Let $T : D(T) \subset X \to X$ be a $\varphi-$strongly quasi-accretive operator. Suppose T is locally bounded at its zero $q \in N(T)$. Then there exists a closed ball B_3 such that the Ishikawa iterative sequence $\{x_n\}_{n\geq 0}$ defined by*

$$(\mathrm{IS})_3 \begin{cases} x_0 \in B_3 \\ x_{n+1} = x_n - \alpha_n T y_n, n \geq 0, \\ y_n = x_n - \beta_n T x_n, n \geq 0, \end{cases}$$

remains in B_3 and converges strongly to q provided that $\{\alpha_n\}$ and $\{\beta_n\}$ are two real sequences in $[0,1)$ satisfying the following conditions:

(i) $\alpha_n \leq \min\{\tau, \frac{r}{2M}\}, n \geq 0;$

(ii) $\beta_n \leq \min\{\tau, \frac{\delta}{M}, \frac{r}{4M}\}, n \geq 0;$

(iii) $\sum_{n=0}^{\infty} \alpha_n = \infty$;

(iv) $\alpha_n \to 0$, $\beta_n \to 0$ *as* $n \to \infty$, *where* r, τ, M *and* δ *are some fixed constants.*

Proof We observe first that there exists a positive number $r > 0$ such that $B_3 = \{x \in D(T) : \|x - q\| \le r\}$ is contained in $D(T)$ and $T(B_3)$ is bounded. Set $M = \sup\{\|Ty\| : y \in B_3\} + 1$ and $M_1 = \max\{r, 1\} M^2$. Observing that b is a continuous and nondecreasing function, we can choose the largest number $\tau > 0$ such that $b(\tau) \le \frac{r\varphi(\frac{r}{4})}{4M_1}$. This is possible, since $b(t) \to 0$ as $t \to 0$. Since X is uniformly smooth, j is uniformly continuous on bounded subsets of X. Therefore for some $\varepsilon = \frac{r\varphi(\frac{r}{4})}{8M}$, we can choose $\delta > 0$ such that $\|jx - jy\| \le \varepsilon$ whenever $\|x - y\| \le \delta$ provided that for all $x, y \in B_r(0)$. Now we can choose $\{\alpha_n\}$ and $\{\beta_n\}$ satisfying conditions (i) and (ii). The rest follows as in the proof of Theorem 2.1, and this completes the proof of Theorem 2.2. □

Corollary 2.3 *Let* $T : D(T) \subset X \to X$ *be a* $\varphi-$*strongly accretive operator with an open domain* $D(T)$. *Suppose* T *has a zero* $q \in N(T)$. *Let* $\{\alpha_n\}$ *and* $\{\beta_n\}$ *be as in Theorem 2.2. Then the conclusion of Theorem 2.2 holds true.*

Proof We observe that T is locally bounded at $q \in D(T)$ and $\varphi-$strongly quasi-accretive. Now the conclusion exactly follows from Theorem2.2. □

Corollary 2.4 *Let* $T : D(T) \subset X \to X$ *be a* $\varphi-$*strongly quasi-accretive operator. Suppose* φ *is surjective and* $D(T)$ *contains a closed ball* $B_4 = \{x \in D(T) : \|x - x_0\| \le 3\varphi^{-1}(\|Tx_0\|)\}$ *such that* $T(B_4)$ *is bounded for some initial value* $x_0 \in D(T)$. *Then the Ishikawa iterative sequence* $\{x_n\}_{n \ge 0}$ *generated by*

$$(\mathrm{IS})_4 \begin{cases} x_0 \in B_4 \\ x_{n+1} = x_n - \alpha_n T y_n, n \ge 0, \\ y_n = x_n - \beta_n T x_n, n \ge 0, \end{cases}$$

remains in B_4 *and converges strongly to the unique zero of* T *provided that* $\{\alpha_n\}$ *and* $\{\beta_n\}$ *are two real sequences in* $[0,1)$ *satisfying the following conditions:*

(i) $\alpha_n \le \min\{\tau, \frac{\varphi^{-1}(\|Tx_0\|)}{4M}\}, n \ge 0$;

(ii) $\beta_n \le \min\{\tau, \frac{\delta}{2M}, \frac{\varphi^{-1}(\|Tx_0\|)}{4M}\}, n \ge 0$;

(iii) $\sum_{n=0}^{\infty} \alpha_n = \infty$;

(iv) $\alpha_n \to 0$, $\beta_n \to 0$ *as* $n \to \infty$, *where* τ, M *and* δ *are some fixed constants.*

Proof Since T is $\varphi-$strongly quasi-accretive, we know that T has a unique zero point in $D(T)$, say, q. Set $r = 2\varphi^{-1}(\|Tx_0\|)$. Then $B_r = \{x \in D(T) : \|x - q\| \le r\}$ is contained in B_4. Now the conclusion of Corollary 2.4 follows from Theorem 2.2. □

References:

[1] BARBU V. *Nonlinear Semigroups and Differential Equations in Banach Spaces* [M]. Noordhoff Int. Publ., Leyden, The Netherlands, 1976.

[2] BROWDER F E. *Nonlinear operators and nonlinear equations of evolution in Banach spaces* [J]. Proc. Sympos. Pure. Math., 1976, **18**(2):

[3] CHIDUME C E. *Iterative solution of nonlinear equations with strongly accretive operators* [J]. J. Math. Anal. Appl., 1995, **192**: 502–518.
[4] CHIDUME C E. *Iterative solutions of nonlinear equations in smooth Banach spaces* [J]. Nonl. Anal., 1996, **26**: 1823–1834.
[5] CHIDUME C E, MOORE C. *The solution by iteration of nonlinear equations in uniformly smooth Banach spaces* [J]. J. Math. Anal. Appl., 1997, **215**: 132–146.
[6] DEIMLING K. *Nonlinear Functional Analysis* [M]. Springer-Verlag, New York/Berlin, 1985.
[7] DEIMLING K. *Zeros of accretive operators* [J]. Manuscripta Math., 1974, **13**: 365–374.
[8] DENG L, DING X P. *Iterative approximation of Lipschitz strictly pseudocontractive mappings in uniformly smooth Banach spaces* [J]. Nonlinear Anal., 1995, **24**: 981–987.
[9] FITZPATRICK P M, HESS P, KATO T. *Local boundedness of monotone type operators* [J]. Proc.Japan Acad., 1972, **48**: 275–277.
[10] KATO T. *Nonlinear semigroups and evolution equations* [J]. J. Math. Soc. Japan, 1967, **18/19**: 508–520.
[11] LIU L S. *Ishikawa and Mann iteration process with errors for nonlinear strongly accretive mappings in Banach spaces* [J]. J. Math. Anal. Appl., 1995, **194**: 114–125.
[12] OSILKE M O. *Iterative solution of nonlinear equations of the Φ-strongly accretive type* [J]. J. Math. Anal. Appl., 1996, **200**: 259–271.
[13] REICH S. *An iterative procedure for constructing zeros of accretive sets in Banach spaces* [J]. Nonl.Anal., 1978, **2**: 85–92.
[14] XU Z B, ROACH G F. *A necessary and sufficient condition for convergence of steepest descent approximation to accretive operator equations* [J]. J. Math. Anal. Appl., 1991, **167**: 189–210.
[15] ZHOU H Y, JIA Y T. *Approximation of fixed points of strongly pseudocontractive maps without Lipschitz assumption* [J]. Proc. Amer. Math. Soc., 1997, **125**: 1705–1709.
[16] ZHOU H Y, JIA Y T. *Approximating zeros of accretive operators by the Ishikawa iteration process* [J]. Abstr. Appl. Anal., 1996, **1**(2): 153–167.
[17] ZHOU H Y. *Iterative solutions of nonlinear equations involving strongly accretive operators without Lipschitz assumption* [J]. J. Math. Anal. Appl., 1997, **213**: 296–307.
[18] ZHOU H Y. *A note on a theorem of Xu and Roach* [J]. J. Math. Anal. Appl., 1998, **227**: 300–304.
[19] ZHOU H Y, CHEN D Q. *Iterative solution of nonlinear equations involving $\varphi-$quasi-accretive operators without Lipschitz assumption* [J]. Math. Sci. Res., Hot-Line, 1999, **3**: 15–26.
[20] ZHOU H Y, CHO Y J. *Ishikawa and Mann iterative processes with errors for nonlinear $\varphi-$strongly quasi-accretive mappings in normed linear spaces* [J], J. Korean Math. Soc., 1999, **36**: 1061–1073.

没有 Lipschitz 假设的 φ- 半压缩映象的不动点与 φ- 强拟增生算子方程解的逼近

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摘 要: 设 X 为实一致光滑 Banach 空间, $T: D(T) \subset X \to X$ 为 φ- 半压缩映象且在它的不动点 q 处是局部有界的. 本文证明了 Mann 迭代与 Ishikawa 迭代过程强收敛于 T 的唯一不动点 q. 几个相关的结果处理 φ- 强拟增生算子方程的解的迭代构造. 本文所得到的结果扩展并推广了 Xu 和 Roach, Zhou 和 Jia 等人的相应结果.