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Some Random Coincidence Point Theorems *

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Abstract: A class of multifunctions satisfying more general contractive inequalities is introduced and random coincidence point theorems for pairs of measurable multifunctions belonging to this class are proved.

Key words: separable metric space; multifunctions; random coincidence point.

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1. Introduction

During the last twenty years many results concerning random coincidence points of various types of operators have been obtained and a number of their applications have been given. Hence, it is necessary to prove a random coincidence point theorem under very mild conditions that includes most of the known results and which is applicable to random differential equations, random integral equations, random approximations etc. In order to generalize the well known contraction principle of Banach to multivalued functions and random fixed point theorems, many authors ([2], [5], [7], and [9]) introduce more general contractive inequalities. We intend to consider a class of generalized contractions that includes the classes considered in [2], [5], [9] and that enables us to prove a more general random coincidence point theorems for multifunctions.

2. Preliminaries

Through out this paper (X,d) is a separable complete metric space and $(\Omega,\ \delta)$ is measurable space. Let 2^X be the family of all subsets of X, $CB(X)$ denotes the family of all nonempty closed bounded subsets of X.

Definition 2.1 *A mapping $\mu:\Omega\to 2^X$ is called measurable if for any open subset C of X, $\mu^{-1}(C)=\{w\in\Omega:\mu(w)\cap C\neq 0\}\in\delta$. This type of measurability is usually called weakly measurable (cf. [4]), but in this paper since we only use this type of measurability,*

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we omit the term "weakly" for simplicity.

Definition 2.2 *A mapping $\xi : \Omega \to X$ is said to be measurable selector of a measurable mapping $\mu : \Omega \to 2^X$ if μ is measurable and for any $w \in \Omega, \xi(w) \in \mu(w)$.*

Definition 2.3 *A mapping $f : \Omega \times X \to X$ is called a random operator if for any $x \in X, f(.,x)$ is measurable. A mapping $T : \Omega \times X \to CB(X)$ is called a multifunction if for every $x \in X, T(.,x)$ is measurable.*

Definition 2.4 *A measurable mapping $\xi : \Omega \to X$ is called a random fixed point of a multifunction (random operator) $T : \Omega \times X \to CB(X)(f : \Omega \times X \to X)$ if for every $w \in \Omega, \xi(w) \in T(w,\xi(w))(f(w,\xi(w)) = \xi(w))$. A measurable mapping $\xi : \Omega \to X$ is a random coincidence point of $T : \Omega \times X \to CB(X)$ and $f : \Omega \times X \to X$ if for every $w \in \Omega, f(w,\xi(w)) \in T(w,\xi(w))$. We denote $\Pi(T)$ the set of random coincidence points of T. Mappings $T : X \to CB(X), f : X \to X$ are compatible if, whenever there is a sequence $\{x_n\}$ in X satisfying $lim_n fx_n \in lim_n Tx_n$ (provided $lim_n fx_n$ exists in X and $lim_n Tx_n$ exists in $CB(X)$) then $lim_n H(fTx_n, Tfx_n) = 0$, where H is a Hausdorff metric on $CB(X)$ induced by the metric d of X; that is, for A, B in $CB(X)$,*

$$H(A,B) = \max\{\sup_{a\in A} P(a,B), \sup_{b\in B} P(b,A)\},$$

where $P(x,E)$ is the distance from a point $x \in X$ to a subset $E \subset X$, i.e., $P(x,E) = \inf\{d(x,y) : y \in E\}$. Random operators $f : \Omega \times X \to X$ and T: $\Omega \times X \to CB(X)$ are compatible if $f(w,.)$ and $T(w,.)$ are compatible for each $w \in \Omega$.

For the remaining part of this section $S, T : \Omega \times X \to CB(X)$ are multifunctions, $f : \Omega \times X \to X$ is random operator and $\xi_n : \Omega \to CB(X)$ is measurable mapping for each $n = 0, 1, 2, \cdots$.

Definition 2.5 *For a map $\xi_0 : \Omega \to X$ if there exists a sequence $\{\xi_n(w)\}$ such that $f(w,\xi_{n+1}(w)) \in S(w,\xi_n(w)), f(w,\xi_{n+2}(w)) \in T(w,\xi_{n+1}(w)), n = 0,1,2,\cdots$, then $O_f(\xi_0(w))$ = $\{f(w,\xi_n(w))$: $n = 1,2,3,\cdots$, for each $w \in \Omega\}$ is the orbit for (S,T,f) at $\xi_0(w)$. $O_f(w,\xi_0(w))$ is called a regular orbit for (S,T,f) if for each n, for each $w \in \Omega$,*

$$d(f(w,\xi_{n+1}(w)), f(w,\xi_{n+2}(w))) \leq H(S(w,\xi_n(w)), T(w,\xi_{n+1}(w))).$$

Definition 2.6 *If there exists a measurable map $\xi : \Omega \to X$ such that $f(w,\xi_n(w)) \to f(w,\xi(w))$ for all $w \in \Omega$, then $O_f(\xi_0(w))$ converge in X. If $O_f(\xi_n(w))$ converge in X, then X is called $(S,T,f,\xi_0(w))$-orbitally complete.*

For every $\beta > 0$, the measurable mapping $g(.,\beta) : \Omega \to (0,1)$ is said to have property (Q): if for $s > 0$, there exists measurable maps $\alpha(.,s) : \Omega \to (0,\infty)$ and $F(.,s) : \Omega \to (0,1)$ such that for every $w \in \Omega, 0 \leq r - s < \alpha(w,s)$ implies $g(w,r) \leq F(w,s)$.

Definition 2.7 *A pair (S,T) of multivalued random operators is said to be asymptotically regular with respect to $f(w,.)$ at $\xi_0(w)$, if for each sequence $\{\xi_n(w)\}, f(w,\xi_{n+1}(w)) \in S(w,\xi_n(w))$, $f(w,\xi_{n+2}(w)) \in T(w,\xi_{n+1}(w)))$, $\lim_n d(f(w,\xi_{n+1}(w)), f(w,\xi_{n+2}(w))) = 0$*

for each $w \in \Omega$. The sequence $\{f(w, \xi_n(w))\}$ is said to be asymptotically $T(w,.)$-regular with respect to $f(w,.)$ if $P(f(w,\xi_n(w)), T(w,\xi_n(w))) \to 0$, for each $w \in \Omega$.

Consider the set R of all continuous functions $g : [0,\infty)^5 \to [0,\infty)$ satisfying the following conditions:

(i) $g(1,1,1,2,0) = g(1,1,1,0,2) = h \in (0,1)$;

(ii) $g(\alpha x_1, \alpha x_2, \alpha x_3, \alpha x_4, \alpha x_5) \le \alpha g(x_1, x_2, x_3, x_4, x_5)$ for $x_i \in [0,\infty), i = 1,5$ and $\alpha \ge 0$;

(iii) if $x_i, y_i \in [0,\infty), x_i < y_i$ for i=1, 4, we have $g(x_1,x_2,x_3,x_4,0) < g(y_1,y_2,y_3,y_4,0)$ and $g(x_1,x_2,x_3,0,x_4) < g(y_1,y_2,y_3,0,y_4)$.

3. Main Results

In this section we give stochastic version of results of Hideaki, Kaneko ([3], Theorem 2) and singh ([10], Theorem 4).

Let $S, T : \Omega \times X \to CB(X)$ be two multifunctions and let $f : \Omega \times X \to X$ be a random operator such that,

$$\begin{aligned} H(S(w,x), T(w,y)) \le & \alpha(w) \max\{d(f(w,x), f(w,y)), P(f(w,x), S(w,x)), P(f(w,y), T(w,y)), \\ & [P(f(w,x), T(w,y)) + P(f(w,y), S(w,x))]/2\} \cdots \qquad (*) \end{aligned}$$

for all $x, y \in X$ and for all $w \in \Omega$, where $\alpha : \Omega \to (0,1)$ is measurable map.

Theorem 3.1 *Let $S, T : \Omega \times X \to CB(X)$ be two continuous multifunctions and let $f : \Omega \times X \to X$ be a random operator such that $S(w,X) \cup T(w,X) \subseteq f(w,X)$ and for a measurable map $\xi_0 : \Omega \to X, f(w,X)$ is $(S,T,f,\xi_0(w))$- orbitally complete, for every $w \in \Omega$, and satisfy (*) for all $w \in \Omega$ and $x, y \in X$. Then there exists a random coincidence point of S,T and f.*

Proof Let $\beta(w) = \sqrt{\alpha(w)}$ for each $w \in \Omega$. Let $\xi_0 : \Omega \to X$ be a measurable mapping and $y_0(w) = f(w, \xi_0(w))$. Let $\xi_1 : \Omega \to X$ be a measurable mapping such that $y_1 : \Omega \to X$ defined by $y_1(w) = f(w,\xi_1(w)) \in S(w,\xi_0(w))$, for all $w \in \Omega$. Indeed, since S is a continuous random operator, we conclude that, for every $v \in X$, the map $(w,x) \to P(v, S(w,x))$ is a caratheodory function (that is measurable in $w \in \Omega$, continuous in $x \in X$). Thus it is jointly measurable. Hence, since $\xi_0 : \Omega \to X$ is measurable, $w \to P(v, S(w, \xi_0(w))$ is measurable too, therefore $w \to S(w,\xi_0(w))$ is weakly measurable by Wagner ([11], p868). By Kuratowski, K ([8], selection Theorem 8), there exists a measurable map $\xi_1 : \Omega \to X$ such that $y_1(w) = f(w,\xi_1(w)) \in S(w,\xi_0(w))$ for all $w \in \Omega$. It further implies by, Papageorgiou ([9], Lemma 2.3) and the fact that $T(w,X) \subseteq f(w,X)$ for every $w \in \Omega$. There exists a measurable mapping $\xi_2 : \Omega \to X$ such that, for each $w \in \Omega, y_2(w) = f(w,\xi_2(w)) \in T(w,\xi_1(w))$ and

$$\begin{aligned} & d(f(w,\xi_1(w)), f(w,\xi_2(w))) \le [1/\beta(w)] H(S(w,\xi_0(w)), T(w,\xi_1(w))) \\ & \le \beta(w) \max\{d(f(w,\xi_0(w)), f(w,\xi_1(w))), d(f(w,\xi_1(w)), f(w,\xi_2(w))), \\ & [d(f(w,\xi_0(w)), f(w,\xi_1(w))) + d(f(w,\xi_1(w)), f(w,\xi_2(w)))]/2\}. \end{aligned}$$

If there exists some $w \in \Omega$ such that

$$d(f(w,\xi_1(w)), f(w,\xi_2(w))) \ge d(f(w,\xi_0(w)), f(w,\xi_1(w)))$$

then

$$d(f(w,\xi_1(w)),f(w,\xi_2(w)))\le\beta(w)d(f(w,\xi_1(w)),f(w,\xi_2(w)))$$

a contradiction, since $\beta(w)<1$, thus for every $w\in\Omega$.

$$d(f(w,\xi_1(w)),f(w,\xi_2(w)))\le\beta(w)d(f(w,\xi_0(w)),f(w,\xi_1(w)))$$

By induction, we produce a sequence of measurable mapping $\xi_n:\Omega\to X$ such that, for any $k\ge 0$, and any $w\in\Omega, f(w,\xi_{2k+1}(w))\in S(w,\xi_{2k}(w)), f(w,\xi_{2k+2}(w))\in T(w,\xi_{2k+1}(w))$ and

$$\begin{aligned}d(f(w,\xi_n(w)),f(w,\xi_{n+1}(w)))\le&\beta(w)d(f(w,\xi_{n-1}(w)),f(w,\xi_n(w)))\\ &\le\ldots\le\beta^n(w)d(f(w,\xi_0(w)),f(w,\xi_1(w))).\end{aligned}$$

Further more, for $m>n$,

$$d(f(w,\xi_n(w)),f(w,\xi_m(w)))\le\{\beta^n(w)+\beta^{n+1}(w)+\ldots+\beta^{m-1}(w)\}d(f(w,\xi_0(w)),f(w,\xi_1(w))).$$

It follows that, for any $w\in\Omega,\{f(w,\xi_n(w))\}$ is a cauchy sequence in $f(w,X)$. The orbital completeness of $f(w,X)$ allows us to obtain a measurable map $\xi:\Omega\to X$ such that $f(w,\xi_n(w))\to f(w,\xi(w))$ for all $w\in\Omega$. It further implies that $f(w,\xi_{2k+1}(w))\to f(w,\xi(w))$ and $f(w,\xi_{2k+2}(w))\to f(w,\xi(w))$. Thus we have, for any $w\in\Omega$,

$$\begin{aligned}&P(f(w,\xi(w)),S(w,\xi(w)))\\ &\quad\le d(f(w,\xi(w)),f(w,\xi_{2k+2}(w)))+P(f(w,\xi_{2k+2}(w)),S(w,\xi(w)))\\ &\quad\le d(f(w,\xi(w)),f(w,\xi_{2k+2}(w)))+H(T(w,\xi_{2k+1}(w)),S(w,\xi(w)))\\ &\quad\le d(f(w,\xi(w)),f(w,\xi_{2k+2}(w)))+\alpha(w)max\{d(f(w,\xi_{2k+1}(w)),f(w,\xi(w))),\\ &\quad\ P(f(w,\xi(w)),S(w,\xi(w))),d(f(w,\xi_{2k+1}(w)),f(w,\xi_{2k+2}(w))),\\ &\quad[d(f(w,\xi(w)),f(w,\xi_{2k+2}(w)))+P(f(w,\xi_{2k+1}(w)),S(w,\xi(w)))]/2\}.\end{aligned}$$

Letting $k\to\infty$, we have

$$P(f(w,\xi(w),S(w,\xi(w)))\le\alpha(w)P(f(w,\xi(w)),S(w,\xi(w)))$$

since $\alpha(w)<1$, we conclude that $f(w,\xi(w))\in S(w,\xi(w))$, for each $w\in\Omega$. Similarly, $f(w,\xi(w))\in T(w,\xi(w))$, for each $w\in\Omega$.

The following theorem is the stochastic version of Singh ([10], Theorem 4).

Theorem 3.2 *Let X be a separable compact metric space and let $S_n,T_n:\Omega\times X\to CB(X)$ be continuous multifunctions which converges pointwise to the functions $S,T:\Omega\times X\to CB(X)$ and let $f:\Omega\times X\to X$ be a random operator such that $S(w,X)\cup T(w,X)\subseteq f(w,X)$ and $f(w,X)$ is compact for every $w\in\Omega$. If there exists a measurable mapping $\alpha:\Omega\to(0,1)$ such that for all $x,y\in\Omega$,*

$$\begin{aligned}H(S_n(w,x),T_n(w,x))\le&\alpha(w)\max\{d(f(w,x),f(w,y)),P(f(w,x),S_n(w,x)),P(f(w,y),T_n(w)),\\ &[P(f(w,y),S_n(w,x))+P(f(w,x),T_n(w,y))]/2\}.\end{aligned}$$

Then there exists a random coincidence point of S, T and f.

Proof We can easily prove this theorem by applying theorem 3.1 and using the approach of Singh[10].

Remark 3.1 If, in (*), each of the terms $P(f(w,x),T(w,y))$ and $P(f(w,y),S(w,x))$ is replaced by $1/2[P(f(w,x),T(w,y))+P(f(w,y),S(w,x))]$, we obtain stochastic version of Theorem 4 of Hideaki, Kaneko[3].

Let $S,T:\Omega\times X\to CB(X)$ be two multifunctions and $f:\Omega\times X\to X$ be random operator such that

$$\begin{aligned} H(S(w,x),T(w,y)) \leq & g_w(d(f(w,x),f(w,y)),P(f(w,x),S(w,x)),P(f(w,y),T(w,y)),\\ & P(f(w,x),T(w,y)),P(f(w,y),S(w,x)))\cdots \qquad (**) \end{aligned}$$

for all $w\in\Omega$ and for all $x,y\in X$ for some $g_w\in R$, such that $h:\Omega\to(0,1)$, $h(w)=g_w(1,1,1,2,0)$ is a measurable function.

Lemma 3.1 *If $S,T:\Omega\times X\to CB(X)$ are multifunctions and $f:\Omega\times X\to X$ be random operator satisfying (**) then $\Pi(S)=\Pi(T)$ and for $f(w,\xi(w))\in\Pi(S)=\Pi(T)$ we have $S(w,\xi(w))=T(w,\xi(w))$ for all $w\in\Omega$. where $\xi:\Omega\to X$ is measurable map.*

Proof Let $f(w,\xi(w))\in\Pi(S)$. we have then $f(w,\xi(w))\in S(w,\xi(w)), w\in\Omega$, and we deduce

$$\begin{aligned} P(f(w,\xi(w)),T(w,\xi(w))) &\leq H(S(w,\xi(w)),T(w,\xi(w)))\\ &\leq g_w(0,0,P(f(w,\xi(w)),T(w,\xi(w))),0,P(f(w,\xi(w)),T(w,\xi(w)))). \end{aligned}$$

We obtain $P(f(w,\xi(w)),T(w,\xi(w)))=0$, $w\in\Omega$. So that $f(w,\xi(w))\in T(w,\xi(w))$ for all $w\in\Omega$. Hence $\Pi(S)\subseteq\Pi(T)$ similarly we have that $\Pi(T)\subseteq\Pi(S)$ and we conclude thus $\Pi(S)=\Pi(T)$. If $f(w,\xi(w))\in\Pi(S)=\Pi(T)$ we get

$$H(S(w,\xi(w)),T(w,\xi(w)))\leq g_w(0,,0,0,0,0)=0$$

so that $S(w,\xi(w))=T(w,\xi(w))$, for all $w\in\Omega$.

The improved version of Theorem 2.1 of Adrian Constantin[1] for random coincidence point is as follows.

Theorem 3.3 *Let $S,T:\Omega\times X\to CB(X)$ be multifunctions and $f:\Omega\times X\to X$ be random operator such that*

(i) $S(w,.),T(w,.)$ are both continuous for all $w\in\Omega$;

(ii) $S(.,x),T(.,x)$ are both measurable for all $x\in X$;

(iii) S,T,f satisfy (**) for all $w\in\Omega$ and all $x,y\in X$;

(iv) $S(w,X)\cup T(w,X)\subseteq f(w,X)$ and for a measurable map $\xi_0:\Omega\to X, f(w,X)$ is $(S,T,\xi_0(w))$-orbitally complete, for every $w\in\Omega$;

Then S,T and f have the same non empty set of random coincidence points and if $f(w,\xi(w))\in\Pi(S)=\Pi(T)$ we have $S(w,\xi(w))=T(w,\xi(w))$ for all $w\in\Omega$.

Proof We define a sequence of measurable mappings $\xi_n : \Omega \to X$ such that $f(w, \xi_{2n+1}(w)) \in S(w, \xi_{2n}(w))$, $w \in \Omega$, and $f(w, \xi_{2n+2}(w)) \in T(w, \xi_{2n+1}(w))$, $w \in \Omega$, then we obtain,

$$\begin{aligned}
&d(f(w,\xi_{2n+1}(w)), f(w,\xi_{2n+2}(w))) \le [1/\sqrt{h(w)}]H(S(w,\xi_{2n}(w)), T(w,\xi_{2n+1}(w)))\\
&\quad \le [1/\sqrt{h(w)}]g_w(d(f(w,\xi_{2n}(w)), f(w,\xi_{2n+1}(w))), d(f(w,\xi_{2n}(w)), f(w,\xi_{2n+1}(w))),\\
&\quad\ d(f(w,\xi_{2n+1}(w)), f(w,\xi_{2n+2}(w))), d(f(w,\xi_{2n}(w)), f(w,\xi_{2n+1}(w)))+\\
&\quad\ d(f(w,\xi_{2n+1}(w)), f(w,\xi_{2n+2}(w))), 0).
\end{aligned}$$

Since $h(w)[1/\sqrt{h(w)}] = \sqrt{h(w)} < 1$ we obtain by Lemma 1.4 and Lemma 1.3 of Adrian Constantin that,

$$d(f(w,\xi_{2n+1}(w)), f(w,\xi_{2n+2}(w))) \le \sqrt{h(w)}d(f(w,\xi_{2n}(w)), f(w,\xi_{2n+1}(w))).$$

We deduce

$$d(f(w,\xi_{2n}(w)), f(w,\xi_{2n+1}(w))) \le (h(w))^{[n/2]}d(f(w,\xi_0(w)), f(w,\xi_1(w))),$$

where $n \ge 1$. Now $h(w) < 1$, $w \in \Omega$, implies that $\{f(w, \xi_n(w))\}$ is a Cauchy sequence in $f(w, X)$ The orbital completeness of $f(w, X)$ allows us to obtain measurable map $\xi : \Omega \to X$ such that $f(w, \xi_n(w)) \to f(w, \xi(w))$ for all $w \in \Omega$, We have

$$\begin{aligned}
&P(f(w,\xi(w)), S(w,\xi_{2n}(w)))\\
&\quad \le P(f(w,\xi(w)), T(w,\xi_{2n+1}(w))) + H(S(w,\xi_{2n}(w)), T(w,\xi_{2n+1}(w)))\\
&\quad \le d(f(w,\xi(w)), f(w,\xi_{2n+2}(w))) + g_w(d(f(w,\xi_{2n}(w)), f(w,\xi_{2n+1}(w))),\\
&\quad\ P(f(w,\xi_{2n}(w)), S(w,\xi_{2n}(w))), P(f(w,\xi_{2n+1}(w)), T(w,\xi_{2n+1}(w))),\\
&\quad\ d(f(w,\xi_{2n}(w)), f(w,\xi_{2n+1}(w))), 0).
\end{aligned}$$

Letting $n \to \infty$ we get

$$\begin{aligned}
&P(f(w,\xi(w)), S(w,\xi(w)))\\
&\quad \le g_w(0, P(f(w,\xi(w)), S(w,\xi(w))), P(f(w,\xi(w)), T(w,\xi(w))), P(f(w,\xi(w)), T(w,\xi(w))), 0)\\
&\quad \le g_w(P(f(w,\xi(w)), T(w,\xi(w))), P(f(w,\xi(w)), S(w,\xi(w))), P(f(w,\xi(w)), T(w,\xi(w))),\\
&\quad\ P(f(w,\xi(w)), T(w,\xi(w))) + P(f(w,\xi(w)), S(w,\xi(w))), 0).
\end{aligned}$$

By Lemma 1.3 of Adrian Constantin we obtain

$$P(f(w,\xi(w)), S(w,\xi(w))) \le h(w)P(f(w,\xi(w)), T(w,\xi(w))).$$

Similarly we have

$$P(f(w,\xi(w)), T(w,\xi(w))) \le h(w)P(f(w,\xi(w)), S(w,\xi(w))).$$

Since $h(w) < 1$, for all $w \in \Omega$, we obtain $P(f(w, \xi(w)), S(w, \xi(w))) = 0$ for all $w \in \Omega$. Hence $\Pi(S) \ne 0$ and from Lemma 3.1 we have that $S(w, \xi(w)) = T(w, \xi(w))$ for all $w \in \Omega$ and $f(w, \xi(w)) \in \Pi(S) = \Pi(T) \ne 0$. Hence the proof.

Theorem 3.4 *Let $T : \Omega \times X \to CB(X)$ be multifunction and let $f : \Omega \times X \to X$ be a continuous random operator such that $T(w, X) \subseteq f(w, X)$ for every $w \in \Omega$. If f and T are compatible and for all $x, y \in X$ and $w \in \Omega$*

$$H(T(w,x),T(w,y)) < g(w,d(f(w,x),f(w,y)))d(f(w,x),f(w,y)) \qquad (***)$$

where $g(.,r) : \Omega \to (0,1)$ for every $r > 0$, has property (Q). Then there exists a sequence $\{\xi_n(w)\}$ of measurable mappings which is asymptotically $T(w,.)$-regular with respect to $f(w,.)$ and $f(w, \xi_n(w))$ converges to a random coincidence point of f and T.

Proof We define two sequences of measurable mappings such that for any $w \in \Omega$ and $n > 0$, $y_n(w) = f(w, \xi_n(w)) \in T(w, \xi_{n-1}(w))$. Further, for each $w \in \Omega$,

$$\begin{aligned} d(y_{n+1}(w), y_{n+2}(w)) &= d(f(w,\xi_{n+1}(w)), f(w,\xi_{n+2}(w))) \\ &< g(w, d(f(w,\xi_n(w)), f(w,\xi_{n+1}(w))))d(f(w,\xi_n(w)), f(w,\xi_{n+1}(w))) \\ &< d(f(w,\xi_n(w)), f(w,\xi_{n+1}(w))) = d(y_n(w), y_{n+1}(w)). \end{aligned}$$

It follows that the sequence $\{d(y_n(w), y_{n+1}(w))\}$ is decreasing and converges to its greatest lower bound which we denote by s. Now $s \geq 0$, in fact, $s = 0$. Otherwise by property (Q) of g there exists measurable mappings $\alpha(.,s) : \Omega \to (0,\infty), F(.,s) : \Omega \to (0,1)$, such that $0 \leq r - s < \alpha(w,s)$ implies $g(w,r) < F(w,s)$. For $\alpha(w,s) > 0$, there exists a natural number N such that, whenever $n > N$,

$$d(y_n(w), y_{n+1}(w)) - s < \alpha(w,s).$$

Hence,

$$g(w, d(y_n(w), y_{n+1}(w))) \leq \alpha(w,s).$$

Let, for each $w \in \Omega$,

$$\sigma(w) = \max\{g(w,d(y_0(w),y_1(w))), g(w,d(y_1(w),y_2(w))), \ldots, g(w,d(y_{N-1}(w),y_N(w))), S(w,$$

For $n = 1, 2 \ldots,$

$$\begin{aligned} d(y_n(w), y_{n+1}(w)) &< g(w, d(y_{n-1}(w), y_n(w)))d(y_{n-1}(w), y_n(w)) \\ &\leq \sigma(w)d(y_{n-1}(w), y_n(w)) \leq \ldots \leq \sigma(w)^n d(y_0(w), y_1(w)) \to 0 \end{aligned}$$

as $n \to \infty$ which contradicts the assumption that $s > 0$. Consequently $lim_n d(y_n(w), y_{n+1}(w))$ $= 0$ which implies that

$$\lim_n P(f(w,\xi_n(w)), T(w,\xi_n(w))) = 0.$$

Therefore the sequence $\{\xi_n(w)\}$ is asymptotically $T(w,.)$- regular with respect to $f(w,.)$. Assume that $\{f(w,\xi_n(w))\}$ is not Cauchy sequence. Then there exists a positive number t and two subsequences $\{n(i)\}, \{m(i)\}$ of natural numbers with $n(i) < m(i)$ and such that $d(y_{n(i)}(w), y_{m(i)}(w)) \geq t$, $d(y_{n(i)}(w), y_{m(i)-1}(w)) < t$ for i = 1, 2,... Then for each w$\in \Omega$,

$$t \leq d(y_{n(i)}(w), y_{m(i)}(w)) \leq d(y_{n(i)}(w), y_{m(i)-1}(w)) + d(y_{m(i)-1}(w), y_{m(i)}(w)).$$

Letting $i \to \infty$ and using the fact $d(y_{n(i)}(w), y_{m(i)-1}(w)) < t$, we obtain

$$\lim_n d(y_{n(i)}(w), y_{m(i)}(w)) = t.$$

For this $t > 0$, there exist measurable mappings $\alpha(.,t) : \Omega \to (0,\infty)$, $F(.,t) : \Omega \to (0,1)$ such that $0 \leq r - t < \alpha(w,t)$ implies $g(w,r) \leq F(w,t)$. For $\alpha(w,t) > 0$, there exists a natural number N_0 such that $i \geq N_0$ implies

$$0 \leq d(y_{n(i)}(w), y_{m(i)}(w)) - t < \alpha(w,t).$$

Hence, for $i \geq N_0$,

$$g(w, d(y_{n(i)}(w), y_{m(i)}(w))) \leq F(w,t).$$

Thus for each $w \in \Omega$,

$$\begin{aligned}
&d(y_{n(i)}(w), y_{m(i)}(w)) \\
&\quad \leq d(y_{n(i)}(w), y_{n(i)+1}(w)) + d(y_{n(i)+1}(w), y_{m(i)+1}(w)) + d(y_{m(i)+1}(w), y_{m(i)}(w)) \\
&\quad \leq d(y_{n(i)}(w), y_{n(i)+1}(w)) + g(w, d(y_{n(i)}(w), y_{m(i)}(w)))d(y_{n(i)}(w), y_{m(i)}(w)) + \\
&\qquad d(y_{m(i)+1}(w), y_{m(i)}(w)). \\
&\quad \leq d(y_{n(i)}(w), y_{n(i)+1}(w)) + F(w,t)d(y_{n(i)}(w), y_{m(i)}(w)) + d(y_{m(i)+1}(w), y_{m(i)}(w)).
\end{aligned}$$

Letting $i \to \infty$, we get $t \leq F(w,t)t < t$, a contradiction. Thus $\{f(w, \xi_n(w))\}$ is a Cauchy sequence. By completeness of the space, there exists $\gamma(w) \in X$ such that for each $w \in \Omega$, $d(y_n(w), \gamma(w)) \to 0$ as $n \to \infty$. Continuity of f implies that $d(f(w, y_n(w)), f(w, \gamma(w))) \to 0$. It further implies that

$$\begin{aligned}
H(T(w, y_n(w)), T(w, \gamma(w))) <& g(w, d(f(w, y_n(w)), f(w, \gamma(w))))d(f(w, y_n(w)), f(w, \gamma(w))) \\
<& d(f(w, y_n(w)), f(w, \gamma(w))) \to 0.
\end{aligned}$$

By (***) and the fact that $\{f(w, \xi_n(w))\}$ is a Cauchy sequence implies that there exists $\xi(w) \in CB(X)$ such that $T(w, \xi_n(w)) \to \xi(w)$. (By Itoh ([6], Preposition 1), ξ is measurable.) Furthermore for each $w \in \Omega$,

$$d(\gamma(w), \xi(w)) \leq \lim_n H(T(w, \xi_{n-1}(w)), T(w, \xi_n(w))) = 0.$$

Now

$$P(f(w, y_{n+1}(w)), T(w, y_n(w))) \leq H(f(w, T(w, \xi_n(w))), T(w, f(w, \xi_n(w)))).$$

Letting $n \to \infty$, we obtain $P(f(w, \gamma(w)), T(w, \gamma(w))) = 0$. Hence, $f(w, \gamma(w)) \in T(w, \gamma(w))$ for each $w \in \Omega$.

Remark 3.2 Theorem 3.4 is the stochastic version of corollary 2 of Hideaki, Kaneko[3].

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一些随机重合点定理

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摘 要: 本文引入了一种满足更一般的收缩不等式的多重函数类，并证明了属于该类的可测多重函数对的一些随机重合点定理.

关键词: 可分度量空间；多重函数；随机重合点.