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A Note on the Paper "Matrix Valued Rational Interpolants and Its Error Formula" *

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Abstract: The error formula in the paper [1] is found to be not correct, and its right verson is established and proven.

Key words: matrix; rational interpolants; error formula.

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Given a set of distinct real points $\{x_i : i = 0, 1, \cdots n, x_i \in R\}$ and a set of real matrix data $\{A_i : i = 0, 1, \cdots n, A_i = A(x_i) \in R^{d\times d}\}$. By using of the Samelson inverse for matrices and inverse differences, Gu and Chen constructed the following branched continued fraction ([1])

$$R_n(x) = \frac{P_n(x)}{q_n(x)} = B_0 + \frac{x - x_0}{B_1}{}_{+\cdots+}\frac{x - x_n}{B_n}, \tag{1}$$

where

$$B_0 := B_0(x_0),\ B_0(x_i) = A_i,\ i = 1, 2, \cdots n,$$

$$B_\ell := B_\ell(x_0x_1\cdots x_\ell) = \frac{x_\ell - x_{\ell-1}}{B_{\ell-1}(x_0x_1\cdots x_{\ell-2}x_\ell) - B_{\ell-1}(x_0x_1\cdots x_{\ell-1})},\ \ell \geq 2,$$

$q_n(x)$ is a real scalar positive polynomial, $P_n(x)$ is a $d\times d$ matrix-valued polynomial, both of whose degrees do not exceed n, $q_n(x)|||P_n(x)||^2$, and (1) serves $R_n(x_i) = A_i$, $i = 0, 1, \cdots n$. $R_n(x)$ was called generalized Samelson inverse matrix valued rational interpolants(GMRI), and the error formula of $R_n(x)$ was given by the following theorem.

Theorem 1[1] *Suppose $A(x)$ has an $(n+1)$-st derivatives in (a, b), $x_i \in (a, b)$ for $i = 0, 1, \cdots n$, and $R_n(x) = P_n(x)/q_n(x), (q_n(x) > 0)$ is GMRI. Then for any $x \in (a, b)$, there is some $\xi \in (a, b)$, with*

$$A(x) - R_n(x) = \frac{w_n(x)}{(n+1)!q_n(x)}\frac{\mathrm{d}^{n+1}}{\mathrm{d}x^{n+1}}[q_n(x)A(x)]_{x=\xi}, \tag{2}$$

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where

$$w_n(x) = (x - x_0)(x - x_1)\cdots(x - x_n). \tag{3}$$

But (2) is not correct. Suppose $f(x)$ and $g(x)$ both serve Rolle's theorem on $[a,b]$, then, there exist some two points $\xi, \eta \in (a,b)$, such that $f'(\xi) = g'(\eta) = 0$, but in general, $\xi \neq \eta$. For example, $f(x) = \sqrt{1-(x-1)^2}$, $g(x) = \sin \pi x, [a,b] = [0,2]$. It is easy to see that $f'(x)$ has only one zero point $x = 1$ in $(0,2)$, rather $g'(x)$ has two zero points $x = \frac{1}{2}$ and $x = \frac{3}{2}$ in $(0,2)$. If $A(x) = (a_{ij}(x)), R_n(x) = (r_{ij}(x))$, under the condition of theorem 1, there should exist $\xi_{ij} \in (a,b)$, such that

$$a_{ij}(x) - r_{ij}(x) = \frac{w_n(x)}{(n+1)!q_n(x)} \frac{\mathrm{d}^{n+1}}{\mathrm{d}x^{n+1}}[q_n(x)a_{ij}(x)]_{x=\xi},$$

and again in general, $\xi_{ij} \neq \xi_{i'j'}$, when $(i,j) \neq (i',j'), \xi_{ij} \neq \xi_{i'j'}$.

Theorem 2 *Suppose $A(x)$ has an $(n+1)$ st derivatives in (a,b), $x_i \in (a,b)$ for $i = 0,1,\cdots n$, and $R_n(x) = P_n(x)/q_n(x), (q_n(x) > 0)$ is GMRI. Then for $x \in (a,b)$, there exists a matrix $(\xi_{ij})_{d\times d}, \xi_{ij} \in (a,b)$, with*

$$A(x) - R_n(x) = \frac{w_n(x)}{(n+1)!q_n(x)}(\xi_{ij})_{d\times d}. \tag{4}$$

Proof Without losing generality, one only need to prove that

$$a_{ij}(x) - r_{ij}(x) = \frac{w_n(x)}{(n+1)!q_n(x)}\xi_{ij}, \quad \text{for} \quad x \in x_i (i = 0,1,\cdots,n). \tag{5}$$

Let $f(u) = q_n(u)[a_{ij}(u) - r_{ij}(u)] - \frac{w_n(u)}{w_n(x)} q_n(x)[a_{ij}(x) - r_{ij}(x)]$, obviously, $f(x) = 0, f(x_i) = 0, i = 0,1,\cdots,n$, By using of Rolle's theorem $n+1$ times and noticing that $q_n(u)r_{ij}(u)$ is a polynomial whose degree does not exceed n, we derive that there exists $\eta_{ij} \in (a,b)$, such that

$$f^{(n+1)}(\eta_{ij}) = \frac{\mathrm{d}^{n+1}}{\mathrm{d}x^{n+1}}[q_n(x)a_{ij}(x)]_{x=\eta_{ij}} - \frac{(n+1)!}{w_n(x)} q_n(x)[a_{ij}(x) - r_{ij}(x)] = 0,$$

hence $a_{ij}(x) - r_{ij}(x) = \frac{w_n(x)}{(n+1)!q_n(x)} \frac{\mathrm{d}^{n+1}}{\mathrm{d}x^{n+1}}[q_n(x)a_{ij}(x)]_{x=\eta_{ij}}$, and denote $\frac{\mathrm{d}^{n+1}}{\mathrm{d}x^{n+1}}[q_n(x)a_{ij}(x)]_{x=\eta_{ij}}$ by ξ_{ij}, one finally get (5), and the theorem is proven.

References:

[1] GU Chuan-qing, CHEN Zhi-bing. *Matrix valued rational interpolants and their error formula* [J]. Math. Numer. Sinica, 1995, 17(1): 73–77.

“矩阵有理插值及其误差公式”一文的注

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摘 要: 本文对文 [1] 中所给出的错误的误差公式做了修订，并给出了相应的证明.

关键词: 矩阵；有理插值；误差公式.